Mitigating Shortages of Generic Drugs: The Role of Reliability Certification

Robert Swinney, Xiaoyu Wu, Can Zhang*

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Abstract

Problem definition: Generic drugs play a vital role in keeping costs down in the U.S. healthcare system. However, generic drugs have experienced severe shortages due to supply disruptions, caused by a number of factors, including low profit margins and a lack of information available to buyers about manufacturers' reliability, leading manufacturers to underinvest in supply reliability. Shortages result in numerous negative consequences for patients as well as institutional buyers of generic drugs, such as hospital systems, who may be forced to use more expensive alternative drugs when a shortage occurs. The U.S. Food and Drug Administration has recently sought to address generic drug shortages caused by supply disruptions, and one proposal is to implement a certification system that assesses and discloses the reliability of generic drug manufacturers. In this paper, we investigate the effectiveness of such a certification system.

Methodology/results: We propose a game theoretic model that captures the effects of generic drug manufacturer reliability certification on manufacturers, buyers, a group purchasing organization, and patients. We find that certification may motivate manufacturers to improve their reliability; however, this may increase patients' shortage cost once the manufacturers adjust their wholesale prices and the buyers adjust their sourcing strategies in response. Nevertheless, we also identify conditions under which certification can benefit all stakeholders compared to the status quo with no certification. Finally, we find that combining certification with subsidies—where certified reliable manufacturers are subsidized by the government—can further reduce the shortage cost when the subsidy level is sufficiently high, but it may backfire if the subsidy level is too low.

Managerial implications: Our findings offer insights for policymakers and nonprofit organizations in designing reliability certifications to mitigate generic drug shortages, highlighting when and why certification can benefit various stakeholders in the supply chain, while cautioning against unintended consequences arising from its impact on manufacturers' price competition and buyers' sourcing strategies.

Keywords: generic drugs, supply disruptions, reliability certification

1 Introduction

Generic drugs play a vital role in the U.S. healthcare system thanks to their equivalent therapeutic efficacy but lower prices compared to brand-name counterparts (Meredith, 1996; Haas *et al.*, 2005; Rome *et al.*, 2021). They account for about 89% of prescriptions but just 26% of overall medication spending in the U.S. based on 2016 data (Association for Accessible Medicines, 2017).

*Duke University, Fuqua School Business, {robert.swinney@duke.edu, xiaoyu.wu@duke, c.zhang@duke.edu}

Moreover, generic drugs' share of prescriptions in the U.S. increased from 36% in 1994 to 90% in 2021 (Grabowski *et al.*, 2014; FDA, 2022).

Despite their benefits, in recent decades, generic drugs have experienced numerous shortages (Owens, 2024). Over 80% of the drugs in shortage reported by the Food and Drug Administration (FDA) in 2023 were generic drugs, with a significant portion being generic sterile injectables used to treat cancer, infections, severe pain, and other critical conditions (Wosinska, 2024). Since many of these drugs are medically necessary for patients, shortages can lead to suboptimal patient outcomes or even life-threatening situations (Department of Health and Human Services, 2024; Park *et al.*, 2025). Furthermore, shortages of generic drugs lead to higher costs for institutional purchasers such as hospitals and healthcare systems, who often must divert time and resources to manage these shortages and substitute the drugs in shortage with more expensive alternatives (ASHP, 2023).

A primary reason for these shortages is the prevalence of manufacturing quality issues and production disruptions, which accounted for 62% of reported shortages between 2013 and 2017 (FDA, 2020). As noted by the FDA, these issues arise from a combination of causes, most notably the low profit margins of generic drugs (driven by intense price competition following the expiration of patent protection for their brand-name counterparts), complex manufacturing processes (e.g, the production of sterile injectables requires highly controlled environments and rigorous sterility assurance), and the significant costs associated with adopting high quality (i.e., reliable) manufacturing processes (FDA, 2020). These factors lead to a lack of incentives for manufacturers to improve their process reliability. Consequently, when quality issues in manufacturing processes such as contamination occur, manufacturers often need to halt production, address the problems, and wait for FDA reapproval, resulting in prolonged supply disruptions and shortages. On the demand side, buyers of generic drugs, such as hospitals, typically have limited information about the quality of drug manufacturing processes, which makes it difficult to assess a particular manufacturer's reliability. This affects the incentives of manufacturers to improve their reliability in two ways. First, as highlighted by the FDA, this lack of information "does not enable the market to reward drug manufacturers with price premiums for mature quality management" (FDA, 2020). Second, a lack of information about reliability leads buyers to choose suppliers based solely on price, intensifying price competition and further reducing the margins of generic drug manufacturers. Thus, a lack of information regarding the reliability of drug manufacturers further reduces manufacturer incentives to improve reliability, worsening shortages for generic drugs.

Because of these issues, recent federal policy directives, including Executive Order 14017, have emphasized the need for new approaches to improve pharmaceutical supply chain reliability (The White House, 2021b). One promising FDA proposal is to create a *certification system* that credibly verifies and discloses the process reliability of different manufacturers, allowing the market to recognize and reward manufacturers with high reliability levels, thereby incentivizing generic drug manufacturers to adopt high quality, high reliability manufacturing processes (FDA, 2020; The White House, 2021a). Such certification systems have already begun to emerge, including the nonprofit RISC Rating System (End Drug Shortages Alliance, 2021) and the Quality Management Maturity Program currently being developed by the FDA (FDA, 2023b).

While it is clear that a reliability certification system has the potential to eliminate one of the key underlying causes of generic drug shortages—information asymmetry regarding the reliability of drug manufacturers—it also stands to impact industry dynamics by affecting the sourcing decisions of buyers and price competition between manufacturers. Hence, the ultimate effects of a certification system, including whether and when certification might reduce drug shortages and how it may affect the payoffs of various stakeholders in the supply chain, remain unclear. In this paper, we study precisely this issue as we seek to understand the role of reliability certification in a generic drug supply chain using a game theoretic model that takes into account the impact of certification on the decisions of manufacturers and purchasers of generic drugs.

In practice, in order to streamline purchasing and reduce transaction costs, over 95% of the hospitals in the U.S. purchase drugs through group purchasing organizations (GPOs) instead of directly from manufacturers (HIGPA, 2017). In line with this practical scenario, we consider a generic drug supply chain that consists of two competing and *ex ante* identical manufacturers producing the same generic drug, one GPO, and multiple institutional buyers (i.e., hospitals or health systems) that purchase the generic drug through the GPO. With some probability, each manufacturer experiences a production disruption and is unable to deliver the ordered quantity. If a disruption occurs and a buyer is unable to meet demand for the drug, there are consequences to both the buyer and patients. The buyer incurs costs due to, e.g., having to allocate additional time and resources to manage shortages and substitute with more expensive alternative products. To measure the consequences to patients, we introduce a shortage cost, an increasing, convex function of the severity of the shortage (i.e., the difference between demand and realized supply), which represents the reduction in patient welfare resulting from worse outcomes because they must substitute with other drugs that are less effective or have worse side effects, or higher health care costs because they must substitute with more expensive alternatives.

Manufacturers can increase their reliability level by pursuing reliability improvement initiatives. These initiatives, such as adopting more advanced quality management systems to robustly detect vulnerabilities to prevent the occurrence of production disruptions (FDA, 2020), result in a higher production cost and a lower disruption probability. Each manufacturer determines their reliability level and offers a wholesale price for all buyers that purchase the drug through the GPO. Then, each buyer determines an order quantity from each manufacturer through the GPO, which takes a fraction of the manufacturers' revenue as its commission. In the absence of reliability certification, the reliability level of each manufacturer is its private information. With certification, drawing on the FDA's tiered approach in its pilot programs for assessing drug manufacturers' reliability (Maguire *et al.*, 2023), we assume that a manufacturer is certified as high reliability if its reliability meets a certain threshold and is referred to as low reliability otherwise, and this certification status (i.e., high vs. low reliability) is public information. Our model allows us to compare equilibrium reliability and pricing decisions (by manufacturers), sourcing decisions (by buyers), and shortage risk with and without reliability certification. Using this model, we first study the following two research questions: under what circumstances can a certification system help reduce drug shortages, and how does it impact various stakeholders in the supply chain?

We find that, as expected, a certification system can incentivize manufacturers to improve their reliability; however, if the certification threshold for high reliability is not high enough (i.e., minor reliability improvements are disclosed through certification), certification may increase patients' shortage costs once manufacturers adjust their wholesale prices and buyers adjust their sourcing strategies in response. This suggests that certifying incremental reliability improvements can inadvertently backfire, and thus policymakers should consider setting high certification standards and focusing on certifying the adoption of production processes that yield significant reliability improvements. We also show that, although certification results in a lower cost for buyers, it may hurt manufacturer and GPO profits even if certification is free. Nevertheless, we also identify conditions under which certification can benefit all stakeholders, including manufacturers, buyers, the GPO, and patients, compared to the status quo of no certification. This happens when (1) the production cost before reliability improvement is relatively high such that manufacturer profit margins are low, (2) reliability improvement is not too costly, and (3) the certification threshold is high enough. Taken together, these findings imply that a reliability certification can generate significant societal benefits by achieving a Pareto improvement when implemented prudently—by targeting drugs with low profit margins and available cost-effective reliability improvement options, and by setting high certification standards—while also emphasizing the importance of carefully accounting for how certification may alter manufacturers' pricing and buyers' sourcing decisions to avoid unintended consequences.

In addition to certification, another important measure considered in practice to mitigate drug shortages is for the government to provide subsidies or other financial incentives (e.g., tax reductions) to manufacturers with high reliability (see The White House 2021a, p. 243 and Advanced Regenerative Manufacturing Institute 2022, p. 32). Given that certification does not always help reduce shortage costs, a natural next question is whether financial incentives for certified high reliability manufacturers can enhance the effectiveness of a certification system and further reduce shortages. We find that combining certification with subsidies, where certified high reliability manufacturers receive compensation from the government, can help resolve the potential negative aspects of certification and further reduce the shortage cost when the subsidy level is sufficiently high. However, we also show that this approach may backfire by increasing the shortage cost compared to the case with no subsidies if the certification threshold is low and the subsidy level is not high enough—hence, subsidies must be implemented with care.

The remainder of this paper is organized as follows. Section 2 reviews the related literature. Section 3 analyzes a benchmark model with no certification. Section 4 analyzes and compares the equilibrium outcomes with and without certification. Section 5 considers the impact of subsidies for reliability improvement. Section 6 concludes the paper.

2 Related Literature

Our work relates to three main streams of literature: drug shortages, supply chain disruptions, and the role of certifications in supply chains.

Drug shortages have received considerable attention in both the literature and government reports (see, e.g., Kaakeh *et al.*, 2011; Yurukoglu *et al.*, 2017; FDA, 2020; Hernandez *et al.*, 2020; The White House, 2021a; Park *et al.*, 2025). However, these studies have primarily focused on identifying the root causes of drug shortages or quantifying the impact of these shortages, using various methods including surveys, case studies, and econometric analyses. In contrast, only a handful of studies have examined (either analytically or empirically) strategies for mitigating drug shortages. To name a few, Kim & Morton (2015) study the effectivenss of price control policies that cap price increases during shortages in mitigating drug shortages, Jia & Zhao (2017) study the use of contracts that consist of a wholesale price and a failure-to-supply penalty to reduce shortages, Lee *et al.* (2021) and Hotkar & Gupta (2021) study the implications of mandating manufacturers to report manufacturing interruptions that can potentially cause shortages, using analytical and empirical methods, respectively, and Naumov *et al.* (2024) use a simulation model to compare several interventions to mitigate drug shortages, including one that allocates a larger share of the total market demand to manufacturers with lower disruption risks. To the best of our knowledge, this literature has not analyzed how asymmetric information about manufacturer supply reliability affects incentives for reliability improvement. Our work addresses this gap by analyzing how reliability certification—as a means of providing reliability information—can incentivize manufacturers to improve reliability, and, in turn, affect drug shortages and the payoffs of different stakeholders in the supply chain.

Our work also relates to the literature on supply chain disruptions (see, e.g., Tomlin & Wang, 2011, for a detailed review of this literature). This literature has extensively studied mitigation strategies employed by a firm to manage disruption risks when faced with potentially unreliable suppliers. For example, Tomlin (2006) studies a firm's optimal sourcing and inventory strategies when sourcing from two suppliers, one unreliable and the other reliable but more expensive, Babich *et al.* (2007) study a firm's optimal sourcing decisions when it sources from two price-setting suppliers with correlated disruption events, Yang *et al.* (2012) study a buyer's optimal procurement contract when suppliers possess private information about their type in terms of disruption risk, Ang *et al.* (2017) study a firm's disruption mitigation strategies in a multi-tier supply chain, and Nikoofal & Gümüş (2018) compare output-based and action-based incentive mechanisms offered by a buyer to a supplier whose reliability is private information. We contribute to this literature by studying how the presence of a supply reliability certification affects the reliability and pricing decisions of suppliers, the sourcing decisions of buyers, and the resulting shortages in the supply chain. In addition, unlike existing studies in this literature, which typically consider either exogenous or publicly known supplier reliability, our work examines a scenario where two manufacturers endogenously determine both their reliability levels and prices, and in the absence of certification, their reliability is private information to each manufacturer.

Finally, there is a growing literature that examines the role of certifications in supply chains. Within this literature, a set of papers explores the use of sustainability/responsibility certifications for sourcing from suppliers who have adopted sustainable and responsible practices (see, e.g., Chen & Lee, 2017; Murali *et al.*, 2019; Ramchandani *et al.*, 2020; Agrawal & Zhang, 2024). Our work differs from these studies by focusing on disruption risks that can result in shortages of supply, rather than on responsibility violations that may decrease the demand from downstream consumers. Another set of papers examines the use of product quality certifications to motivate quality efforts from suppliers (see, e.g., Hwang *et al.*, 2006; Chen & Deng, 2013; Bondareva & Pinker, 2019; Bian *et al.*, 2022). Our work differs from these studies in two aspects. First, our focus is on disruption risks resulting from poor process quality rather than product quality. Second, while these studies typically consider scenarios with a powerful buyer who offers contracts to a supplier, we focus on a generic drug supply chain where drug manufacturers set prices and engage in price competition.

3 Benchmark Model: No Certification

In this section, we introduce our baseline model of a generic drug supply chain in the absence of a certification system (referred to as the "no certification" model). Two competing manufacturers produce the same generic drug for use during a single selling season. Given that the vast majority of the hospitals in the U.S. purchase drugs through GPOs instead of directly from manufacturers (HIGPA, 2017), we consider a GPO that serves as the sole marketplace for the drug to a pool of N identical institutional buyers (e.g., hospitals). Each manufacturer determines the wholesale price that buyers pay when purchasing through the GPO. In turn, given the wholesale prices for both manufacturers, each buyer chooses whether to purchase the drug from one or both of the manufacturers. For facilitating the transaction, the GPO takes a fraction of the manufacturers' revenue as a commission.

Most drug shortages arise from manufacturing quality issues that lead to delivery failures (FDA, 2020). To capture this, we consider that both manufacturers are subject to the risk of production disruptions. Further, without certification, each manufacturer's reliability level (i.e., the probability of a manufacturer being able to deliver the intended order) is their private information. Accordingly, we formulate a Bayesian game in which (1) each manufacturer first determines their reliability level, (2) each manufacturer sets its wholesale price, and (3) upon observing the wholesale prices, each buyer determines an order quantity from each manufacturer through the GPO. The sequence of events is summarized in Figure 1.

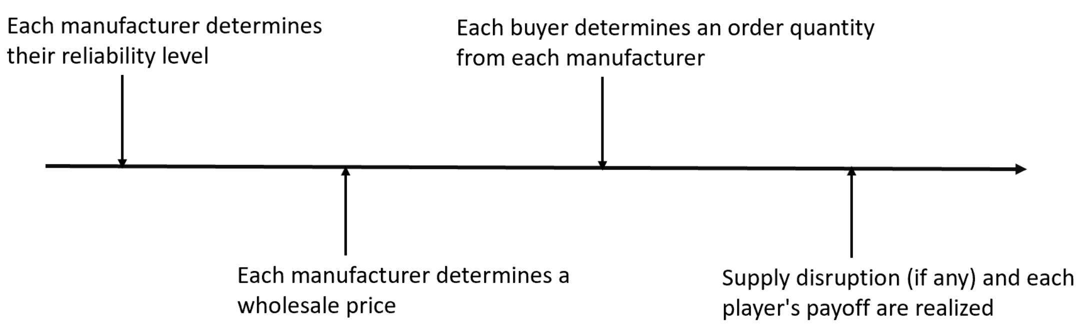

Figure 1. Sequence of events under no certification

Next, we describe the decisions and payoffs of each player in more detail. In the remainder of the paper, we use i, j to label manufacturers and buyers, respectively, where $i \in \{1, 2\}, j \in \{1, ..., N\}$. For $i \in \{1, 2\}$, we use $-i$ to denote manufacturer $3 - i$. Our key notation is summarized in Table 1.

Notation	Description
r_i	Reliability level of manufacturer i, where $r_i \in [r_L, 1]$
$c_i(r_i)$	Unit production cost of manufacturer i, where $c_i(r_i) = c_L + a(r_i - r_L)$
w_i	Wholesale price of manufacturer i
η	GPO's commission rate, $\eta \in [0, 1)$
d	Demand of each buyer
D	Total demand of all buyers, where $D = Nd$
p	Unit shortage cost for each buyer
s	Unit salvage value of unused stock for each buyer
Q_{ij}	Sourcing quantity of buyer j from manufacturer i
f	Fixed cost incurred to each manufacturer to be certified

Table 1. Summary of Key Notation

3.1 The Manufacturers

Both manufacturers begin with a baseline unit production cost c_L and reliability level r_L, i.e., with probability r_L production operates normally and with probability $1 - r_L$ a disruption occurs. In pharmaceutical manufacturing, process quality issues typically necessitate the complete shutdown of production lines, which can last extended periods as resolving quality issues requires thorough investigations, corrective actions, and FDA reapproval before production can resume. As a result, disruptions often result in the failure of an entire delivery (Jia & Zhao, 2017; FDA, 2020; Hotkar & Gupta, 2021). Accordingly, similar to Jia & Zhao (2017), we assume that a manufacturer delivers all of the ordered quantity with probability r_L and delivers nothing with probability $1 - r_L$. This baseline reliability level r_L can be interpreted as the reliability achieved when complying with minimum regulatory standards for production processes, such as the Current Good Manufacturing Practices (CGMPs) required for supplying the U.S. marketplace (FDA, 2020).

Manufacturers can go beyond the minimum regulatory standards by undertaking reliability improvement initiatives, such as adopting more advanced quality management systems to better monitor ongoing processes and detect vulnerabilities to prevent production disruptions (FDA, 2020). To model these decisions, we assume each manufacturer i chooses a reliability level $r_i \geq r_L$, which raises the unit production cost to $c_L + a(r_i - r_L)$. In our main model, we consider a linear cost for reliability improvement for ease of exposition. However, we show in Supplemental Appendix C that our insights remain qualitatively similar under an extension with convex costs.[1] In practice, buyers are typically unable to observe whether each manufacturer has adopted more reliable production processes (FDA, 2020, 2023a). Accordingly, we model each manufacturer's reliability level as their private information.

In addition to determining the reliability level, each manufacturer i also determines the wholesale price at which their production is sold via the GPO, denoted by w_i. Manufacturers are only paid for units that are delivered to buyers (i.e., if a disruption occurs and a manufacturer fails to deliver to its customers, it receives no revenue). Consistent with practice, the manufacturers pay a fraction $\eta \in [0, 1)$ of their revenue to the GPO as the GPO's commission; the role of the GPO and its commission fee is discussed further in §3.3.

Let $c_i(r_i) = c_L + a(r_i - r_L)$. Then, for each delivered unit, manufacturer i's net profit is given by $(1 - \eta)w_i - c_i(r_i)$. In the event of a disruption, manufacturers receive no payment and incur no manufacturing costs.[2] Since the reliability levels of the manufacturers are not observable to the buyers, each buyer makes their sourcing decisions based on the wholesale prices. Thus, given the wholesale prices w_1, w_2, we denote the order quantity of buyer j from manufacturer i as $Q_{ij}(w_1, w_2)$. Manufacturer i's expected profit under no certification can then be written as follows:

$$\Pi_i^{\text{Non-certified}}(r_i, w_i | w_{-i}) = \sum_{j=1}^{N} r_i \left((1 - \eta)w_i - c_i(r_i) \right) Q_{ij}(w_1, w_2). \tag{1}$$

Clearly, to ensure a nonnegative profit for each manufacturer, it is sufficient to consider wholesale prices that satisfy $w_i \geq \frac{c_L}{1-\eta}$.

[1] In addition, for ease of analysis and exposition, we do not consider fixed costs for reliability improvement. Since such a fixed cost does not affect manufacturers' price competition or buyers' sourcing decisions, our insights remain qualitatively similar under an extension that includes a fixed cost.

[2] Similar to Kim & Morton (2015) and Hotkar & Gupta (2021), we do not consider "failure-to-supply" penalties on manufacturers when disruption occurs. This is because such penalties are typically difficult to implement in practice due to the low profit margins of generic drugs and the limited number of manufacturers producing each specific generic drug.

3.2 The Buyers

Each of the N buyers faces deterministic demand d (i.e., the total demand is $D = Nd$).[3] After observing the wholesale prices w_1, w_2, each buyer j sources quantity Q_{ij} from manufacturer i. If a disruption occurs at one or more of the manufacturers, a buyer may have insufficient inventory to satisfy its demand, i.e., a shortage may occur. In turn, this may increase the buyer's cost because the buyer needs to allocate additional time and resources to manage these shortages, and it may also be forced to find alternatives that are more expensive than the generic drug they are replacing (Vizient, 2019; ASHP, 2023). We model this by incorporating a linear shortage cost p incurred by each buyer for each unit of unfulfilled demand.[4]

Facing the possibility of costly shortages, buyers may find it optimal to order more than their demand d and, if no disruption occurs, they may receive a larger quantity of drugs than they can use. In practice, such excess inventory might be stored for future use. To capture the potential value of any excess inventory in our single period model, we incorporate a salvage value s for every unit delivered in excess of demand. Since drug products typically have a short shelf life, we assume the salvage value is not enough to cover the production cost of the product, i.e., $s < c_L$.[5]

The (actual) expected cost of buyer j with sourcing quantities Q_{1j}, Q_{2j} from manufacturers 1 and 2, respectively, given wholesale prices w_1, w_2 and manufacturer reliability levels r_1, r_2 is therefore

$$\begin{aligned} g\left((r_1, r_2), (w_1, w_2), (Q_{1j}, Q_{2j})\right) = {} & r_1 r_2 (p(d - Q_{1j} - Q_{2j})^+ + w_1 Q_{1j} + w_2 Q_{2j} - s(Q_{1j} + Q_{2j} - d)^+) \\ & + r_1(1 - r_2)(p(d - Q_{1j})^+ + w_1 Q_{1j} - s(Q_{1j} - d)^+) \\ & + (1 - r_1) r_2 (p(d - Q_{2j})^+ + w_2 Q_{2j} - s(Q_{2j} - d)^+) \\ & + (1 - r_1)(1 - r_2) p d, \end{aligned} \qquad (2)$$

where the first term denotes buyer j's cost when no disruption occurs, the second and third terms

[3] This is a reasonable assumption because many generic drugs subject to disruption and shortage risks, such as those for cancer and chronic disease treatments, have fairly stable demand (Kacik, 2023; Beusekom, 2023).

[4] In Supplemental Appendix B, we consider the case of convex increasing shortage costs for buyers, which accounts for the possibility that the marginal shortage cost may be larger for more significant shortages.

[5] The fact that $s < c_L$ also reflects the reality that health care providers typically have limited financial resources and, therefore, a high inventory carrying cost; hence, any unit of excess inventory results in a net loss for the buyer.

denote buyer j's cost when one manufacturer is disrupted, and the fourth term denotes buyer j's cost when both manufacturers are disrupted.[6] Clearly, if manufacturer i's price w_i is higher than p, buyers will never source from this manufacturer. Thus, it is without loss of generality to consider $w_i < p$.

Without certification, since manufacturer reliability levels are private information, we let $\mathbb{P}_j(r_1, r_2)$ denote the probability measure representing buyer j's belief on the two manufacturers' reliability levels. Then, each buyer determines an order quantity from each manufacturer, Q_{1j} and Q_{2j}, to minimize its expected cost under asymmetric information, which is defined as follows:

$$L_j^{\text{Non-certified}}(Q_{1j}, Q_{2j}|w_1, w_2) = \int_{r_1}\int_{r_2} g\left((r_1, r_2), (w_1, w_2), (Q_{1j}, Q_{2j})\right) d\mathbb{P}_j(r_1, r_2).$$

3.3 The GPO

GPOs primarily generate revenue through a commission charged on manufacturer sales (i.e., a share of manufacturer revenue) and a membership fee charged to participating buyers. Since we do not model the decision of buyers to join the GPO (a long-term, complex decision that depends on far more than a single generic drug), we normalize the membership fee to zero. We model the commission by assuming the GPO is paid a fraction η of each manufacturer's revenue. In other words, for each unit delivered by manufacturer i, the GPO earns ηw_i while the manufacturer earns $(1-\eta)w_i$. Consequently, given the decisions of manufacturers and buyers, the GPO's expected profit can be expressed as follows.

$$\Pi_{GPO} = \sum_{i=1}^{2}\sum_{j=1}^{N} \eta r_i w_i Q_{ij}. \tag{3}$$

In practice, the GPO's commission is determined by numerous factors beyond the scope of our analysis, such as the total volume of all drugs purchased through the GPO and competition among different GPOs. Accordingly, we treat η as exogenous in our model, which is reasonable because the purchasing dynamics of a single generic drug are unlikely to influence the overall commission rate.

[6] Hospitals typically get reimbursed (e.g., by healthcare insurance) after the drug is administered to patients. However, since the reimbursement amount is usually based on fixed rates that are not directly tied to each hospital's actual procurement cost (Murry *et al.*, 2018), our model and analysis can be extended to capture these reimbursements, with our key insights remaining qualitatively similar.

We further note that, while GPOs may have repeated interactions with manufacturers in practice, the vast scope of GPO operations—spanning numerous drugs and manufacturers—limits their ability to assess the reliability of each manufacturer for each drug. As the FDA highlights, the lack of reliability information poses a key challenge that prevents the market from rewarding high reliability manufacturers (FDA, 2020, 2023a). This aligns with our model, where, in the absence of certification, manufacturers' reliability levels remain private information.

3.4 Patients' Shortage Cost

If one or more manufacturers experiences a disruption, buyers may have insufficient inventory to cover their demand. This results in increased costs for the buyers (as described in §3.2) and, potentially, negative consequences for patients. We model the latter via a *shortage cost* function which incorporates both the health and financial impacts of shortages on patients.

To capture the fact that the negative repercussions of drug shortages on patients are, in practice, more severe for larger mismatches between demand and supply, we assume that this function is increasing and convex in the total inventory shortfall, i.e., demand minus the realized supply across all buyers. Convexity may occur because, for instance, with a small shortfall, hospitals may be able to easily treat patients who are well-suited to available alternatives with minimal impact on outcomes or side effects, but for a large shortfall, even patients that are poorly suited to available alternative drugs may be forced to use them, leading to more significant negative impacts on health.

Given this, we define the expected shortage cost for patients (denoted by L_{patient}) as follows:

$$
\begin{aligned}
L_{\text{patient}} = \gamma\Big(& r_1 r_2 \big(\big(D - \sum_{j=1}^{N}(Q_{1j} + Q_{2j})\big)^+\big)^2 \\
&+ r_1(1-r_2)\big(\big(D - \sum_{j=1}^{N} Q_{1j}\big)^+\big)^2 + (1-r_1)r_2\big(\big(D - \sum_{j=1}^{N} Q_{2j}\big)^+\big)^2 \\
&+ (1-r_1)(1-r_2)D^2\Big),
\end{aligned} \tag{4}
$$

where $(\cdot)^+ = \max\{\cdot, 0\}$ and $\gamma > 0$. In (4), the first term represents the case when both manufacturers deliver, the second and third terms represent the two cases when only one manufacturer delivers, and the fourth term represents the case when neither manufacturers delivers.

3.5 Equilibrium under No Certification

Next, we solve for the perfect Bayesian equilibrium (PBE) of the game depicted in Figure 1, which consists of (1) all players' decisions, including the two manufacturers' reliability levels and wholesale prices and buyers' sourcing decisions, and (2) other players' beliefs on each manufacturer's reliability choice. In accordance with the concept of PBE, we consider that all beliefs are consistent with the manufacturers' reliability decisions, and all players' decisions are sequentially rational (i.e., optimal given the player's belief and other players' subsequent decisions). For simplicity and sharpness of results, we focus on pure-strategy equilibria in our analysis.[7] The proofs of all analytical results are included in the Online Appendix.

We start by characterizing each buyer's optimal sourcing quantity from each manufacturer under a given set of wholesale prices w_1, w_2. For each buyer j, let $\mathbb{E}_j[\cdot]$ denote the expectation operator associated with buyer j's belief about the reliability levels of the two manufacturers.

Lemma 1. *Under no certification, given the wholesale prices* $w_1, w_2 \in [\frac{c_L}{1-\eta}, p)$ *and buyer j's belief about the reliability levels of the two manufacturers, buyer j's optimal sourcing quantities from each manufacturer i (denoted by Q^*_{ij}) are as follows:*

(i) If $\mathbb{E}_j[r_i((1-r_{-i})p + r_{-i}s - w_i)] \geq 0$ *and* $\mathbb{E}_j[r_{-i}((1-r_i)p + r_i s - w_{-i})] \geq 0$, $Q^*_{ij} = Q^*_{(-i)j} = d$.

(ii) If $\mathbb{E}_j[r_i((1-r_{-i})p + r_{-i}s - w_i)] \geq 0$ *and* $\mathbb{E}_j[r_{-i}((1-r_i)p + r_i s - w_{-i})] < 0$, $Q^*_{ij} = d, Q^*_{(-i)j} = 0$.

(iii) If $\mathbb{E}_j[r_i((1-r_{-i})p + r_{-i}s - w_i)] < 0$ *and* $\mathbb{E}_j[r_{-i}((1-r_i)p + r_i s - w_{-i})] < 0$, *then:*

(a) If $\mathbb{E}_j[r_i((1-r_{-i})p + r_{-i}s - w_i)] > \mathbb{E}_j[r_{-i}((1-r_i)p + r_i s - w_{-i})]$, $Q^*_{ij} = d, Q^*_{(-i)j} = 0$.

(b) If $\mathbb{E}_j[r_i((1-r_{-i})p + r_{-i}s - w_i)] = \mathbb{E}_j[r_{-i}((1-r_i)p + r_i s - w_{-i})]$, $Q^*_{1j} = \alpha d, Q^*_{2j} = (1-\alpha)d$, *where* $\alpha \in [0,1]$.

Lemma 1 shows that whether each buyer sources from manufacturer i critically depends on the comparison between the wholesale price w_i and a benefit term $(1-r_{-i})p + r_{-i}s$. Specifically, given that each buyer has a deterministic demand d, it is intuitive that each buyer will source at least d units in total. If a buyer has already sourced d units from manufacturer $-i$, the expected benefit from sourcing an *additional* unit from manufacturer i equals the sum of (1) the expected saving of shortage cost when manufacturer $-i$ experiences a disruption (i.e., $(1-r_{-i})p$) and (2) the expected

[7] When there are multiple optimal reliability levels for a manufacturer, we break the tie by choosing the lowest reliability. Similarly, when there are multiple optimal wholesale prices, we break the tie by choosing the lowest price.

salvage value when manufacturer $-i$ delivers successfully (i.e., $r_{-i}s$). Therefore, if w_i is low relative to the benefit (i.e., $\mathbb{E}_j[r_i((1-r_{-i})p + r_{-i}s - w_i)] \geq 0$) for both manufacturers, it is optimal for each buyer to purchase d from *both* manufacturers.

On the other hand, if w_i is low relative to the benefit while w_{-i} is high, it is optimal for each buyer to only purchase d from manufacturer i. If w_i is high relative to the benefit for both manufacturers, it is optimal for each buyer to purchase from the manufacturer that generates a lower expected cost; if both manufacturers generate the same expected cost, each buyer is indifferent between them and we assume that each buyer sources αd from manufacturer 1 and $(1-\alpha)d$ from manufacturer 2. For ease of exposition, we assume $\alpha = \frac{1}{2}$ in the rest of the analysis.

Given Lemma 1, we proceed to analyze each manufacturer's reliability level (denoted by r_1^*, r_2^*) and wholesale price (denoted by w_1^*, w_2^*) in equilibrium under no certification. Intuitively, if the marginal cost for reliability improvement (i.e., a) is sufficiently low, manufacturers have a strong incentive to improve their reliability even if their reliability levels are not observable to buyers, because higher reliability increases the probability of successful delivery, which increases the expected revenue. In this scenario, reliability issues and shortages will not be a significant problem. Hence, in the remainder of the paper, we focus on the case where the marginal cost of reliability improvement is sufficiently *high*. Formally,

Assumption 1. $a \geq \max\{\frac{(1-\eta)((1-r_L^2)p + r_L^2 s)}{r_L}, \frac{(1-\eta)p - c_L}{4-r_L}\}$.

Under Assumption 1, reliability improvement is costly, such that in equilibrium, manufacturers do not have an incentive to increase reliability under no certification, as we will show below. This aligns with the most relevant practical case given our research focus: when manufacturers lack the incentive to pursue high reliability of their own accord, which, in turn, necessitates the need for mechanisms like reliability certification to reduce the risk of drug shortages (FDA, 2020).

Let

$$\bar{\bar{c}}_L(x) = (1-\eta)\left((1-x)p + xs\right), x \in [r_L, 1]. \tag{5}$$

The following lemma characterizes the equilibrium decisions of all players under no certification:

Lemma 2. *Under no certification, the equilibrium decisions of all players are characterized as follows:*

Condition	r_i^*, r_{-i}^*	w_i^*, w_{-i}^*	$Q_{ij}^*, Q_{(-i)j}^*$
$c_L \leq \bar{\bar{c}}_L(r_L)$	r_L, r_L	$(1-r_L)p + r_L s, (1-r_L)p + r_L s$	d, d
$c_L > \bar{\bar{c}}_L(r_L)$	r_L, r_L	$\frac{c_L}{1-\eta}, \frac{c_L}{1-\eta}$	$\frac{1}{2}d, \frac{1}{2}d$

Table 2. Summary of Equilibrium Decisions under No Certification

(i) If $c_L \leq \bar{\bar{c}}_L(r_L)$, then neither manufacturer improves their reliability, the wholesale price of both manufacturers is $(1-r_L)p + r_L s$, and each buyer sources d from each manufacturer.
(ii) If $c_L > \bar{\bar{c}}_L(r_L)$, then neither manufacturer improves their reliability, the wholesale price of both manufacturers is $\frac{c_L}{1-\eta}$, and each buyer sources $\frac{1}{2}d$ from each manufacturer.

Table 2 provides a summary of Lemma 2. As noted previously, Assumption 1 implies that the marginal cost for reliability improvement is sufficiently high such that the manufacturers never improve their reliability under no certification, as shown in Lemma 2.

Under the baseline reliability r_L, given the GPO's commission η, the minimum wholesale price for each manufacturer to maintain a non-negative profit is $\frac{c_L}{1-\eta}$. From the discussion after Lemma1, the expected benefit of a manufacturer to each buyer given that the buyer has already sourced d from the other manufacturer is $(1-r_L)p+r_L s$. Then, Lemma 2 implies that under no certification, if this expected benefit is higher than the minimum wholesale price (i.e., $(1-r_L)p + r_L s \geq \frac{c_L}{1-\eta}$, or equivalently $c_L \leq \bar{\bar{c}}_L(r_L)$), the manufacturers set the wholesale price equal to the expected benefit $(1-r_L)p + r_L s$, and each buyer sources d from both manufacturers. On the other hand, if the expected benefit is lower than the minimum wholesale price (i.e., $(1-r_L)p + r_L s < \frac{c_L}{1-\eta}$, or equivalently $c_L > \bar{\bar{c}}_L(r_L)$), each manufacturer sets their wholesale price to the minimum level. In this case, the buyers are indifferent between the manufacturers and do not have an incentive to source more than d, so each sources $\frac{d}{2}$ from each manufacturer.

4 Impact of Certification

In this section, we study how certification affects generic drug shortages and various stakeholders in the supply chain. In line with the FDA's tiered approach in its pilot programs for assessing drug manufacturers' reliability (see Maguire *et al.* 2023, fig. 1), we assume that under certification, manufacturers are assessed based on reliability thresholds. Specifically, a manufacturer is certified as high reliability if its reliability meets a certain threshold, denoted by r_H, and is referred to as low reliability otherwise. Further, whether a manufacturer is certified as high reliability is public information. Without loss of generality, we assume $p \geq \frac{c_L + a(r_H - r_L)}{1-\eta}$, since otherwise, it is straightforward to show that buyers would never have an incentive to source from a high reliability manufacturer, and hence manufacturers never have incentive to improve reliability. For ease of analysis and exposition, we assume that both manufacturers are required (e.g., by government policy) to undergo reliability assessment and have their certification status (i.e., high vs. low reliability) publicly revealed. In Supplemental Appendix A, we extend our model to consider a scenario where manufacturers endogenously decide whether or not to participate in certification and show that our insights remain qualitatively similar.

The sequence of events under certification are identical to that under no certification (as discussed in §3 and shown in Figure 1), with one key difference: once the manufacturers determine their reliability levels in the first step of the game, whether their reliability level meets the threshold r_H is made public and revealed to all parties—that is, their reliability level is "certified" to be either low ($r_i < r_H$) or high ($r_i \geq r_H$). Because certification requires manufacturers to document their practices, work with regulators on audits and inspections, collect and distribute data to regulators, and increase the visibility into their processes, it is not costless. Thus, we introduce a fixed certification cost $f \geq 0$ that is borne by each manufacturer, representing the financial and time costs associated with certification.

Given that certification provides public information on whether a manufacturer's reliability meets a certain threshold, buyers make their sourcing decisions based not only on the wholesale prices w_1, w_2, but also on the certification status of the two manufactures, denoted by δ_1, δ_2, where $\delta_i = 1$ if $r_i \geq r_H$ and $\delta_i = 0$ otherwise, for $i \in \{1, 2\}$. We denote the sourcing quantity of buyer j from manufacturer i by $Q_{ij}((\delta_1, \delta_2), (w_1, w_2))$. The objective functions of each player are defined similarly to the benchmark model without certification, except that manufacturer i's expected profit is adjusted as follows:

$$\Pi_i^{\text{Certified}}(r_i, w_i | \delta_{-i}, w_{-i}) = r_i \left((1-\eta) w_i - c_i(r_i) \right) Q_{ij} \left((\delta_1, \delta_2), (w_1, w_2) \right) - f.$$

Condition	r_i^*, r_{-i}^*	w_i^*, w_{-i}^*	$Q_{ij}^*, Q_{(-i)j}^*$
$a \geq \bar{\bar{a}}$ and $c_L \leq \bar{\bar{c}}_L(r_L)$	r_L, r_L	$(1-r_L)p + r_L s, (1-r_L)p + r_L s$	d, d
$a \geq \bar{\bar{a}}$ and $c_L > \bar{\bar{c}}_L(r_L)$	r_L, r_L	$\frac{c_L}{1-\eta}, \frac{c_L}{1-\eta}$	$\frac{1}{2}d, \frac{1}{2}d$
$a < \bar{\bar{a}}$	r_H, r_L	$\frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} - \epsilon$,[8] $\frac{c_L}{1-\eta}$	$d, 0$

Table 3. Summary of Equilibrium Decisions under Certification

4.1 Equilibrium Under Certification

Under certification, buyers observe whether each manufacturer's reliability meets the threshold r_H but not the precise reliability level. Thus, we still have a Bayesian game, and we solve for the PBE as in Section 3.5. Given both manufacturers' certification statuses δ_1, δ_2 and wholesale prices w_1, w_2, and the corresponding buyer beliefs on the manufacturers' reliability levels, each buyer's optimal sourcing quantity from each manufacturer follows the same characterization as in Lemma 1. Let

$$\bar{\bar{a}} = \frac{(r_H - r_L)((1-\eta)p - c_L) - r_L\left(\bar{\bar{c}}_L(r_L) - c_L\right)^+}{r_H(r_H - r_L)}.$$

Then, the equilibrium decisions of all players under certification are given in the following lemma:

Lemma 3. *Under certification, the equilibrium decisions of all players are characterized as follows:*
(i) If $a \geq \bar{\bar{a}}$, neither manufacturer improves reliability. Further, if $c_L \leq \bar{\bar{c}}_L(r_L)$ (the threshold defined in Equation (5)), both manufacturers set a wholesale price of $(1-r_L)p+r_L s$, and each buyer sources d from each manufacturer; otherwise (i.e., if $c_L > \bar{\bar{c}}_L(r_L)$), both manufacturers set a wholesale price of $\frac{c_L}{1-\eta}$, and each buyer sources $\frac{1}{2}d$ from each manufacturer.
(ii) If $a < \bar{\bar{a}}$, one manufacturer improves its reliability to r_H and sets wholesale price $\frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H}$ $-\epsilon$, where ϵ is a small positive number, while the other manufacturer remains at low reliability r_L and sets wholesale price $\frac{c_L}{1-\eta}$. Each buyer single sources d from the manufacturer with high reliability.

Table 3 summarizes the insights from Lemma 3. Under certification, if reliability improvement is very costly (i.e., $a \geq \bar{\bar{a}}$), the equilibrium is the same as under no certification: neither manufacturer chooses high reliability. On the other hand, if reliability improvement is less costly (i.e.,

[8] ϵ is a sufficiently small and positive number

$a < \bar{\bar{a}}$), in equilibrium, one of the manufacturers will increase its reliability to r_H to be certified as high reliability, and all buyers exclusively source from this manufacturer even though it sets a higher price than the other, low reliability, manufacturer.[9] Hence, compared to the no certification case, Lemma 3 shows that certification can spur reliability improvement, provided reliability is not too costly.

The intuition behind this result is that, without certification, although higher reliability increases the probability of successful delivery, the benefit of improving reliability remains limited because buyers cannot directly observe or reward it. As a result, the relatively high cost of reliability improvement (i.e., Assumption 1) gives manufacturers no incentive to invest in reliability. On the other hand, with certification, buyers can recognize high reliability manufacturers and are willing to pay higher prices or purchase more from them. As a result, high reliability becomes a profitable option for manufacturers, provided it is not too costly ($a < \bar{\bar{a}}$).

To summarize, Lemma 3 shows how certification can enable the market to "*recognize and reward manufacturers with mature quality management systems*" (FDA, 2020). Based on this result, one may intuitively expect that certification should lead to a more reliable supply of generic drugs and reduce the cost of drug shortages. However, Section 4.2 will show that this is not always the case.

4.2 Comparison: With vs. Without Certification

Next, we compare the equilibrium outcomes with and without certification across four performance measures: patients' expected shortage cost, the manufacturers' expected profit, the buyers' expected cost, and the GPO's expected profit. Recall that when $a \geq \bar{\bar{a}}$, both certification and no certification result in identical equilibrium outcomes (see Tables 2 and 3); hence, we focus in this section on the more interesting case where $a < \bar{\bar{a}}$. Under this condition, certification results in one manufacturer choosing high reliability, while neither manufacturer improves reliability under no certification. In what follows, we use the terms "higher" and "lower" in the weak sense unless specified otherwise.

[9] Recall that we have assumed in Assumption 1 that the cost for reliability improvement (i.e., a) is relatively high to focus on a practical scenario where no manufacturer improves its reliability without certification. Lemma 3 implies that, under this assumption, at most one manufacturer improves its reliability with certification. We later show that under the same assumption, combining certification with subsidies can motivate both manufacturers to improve reliability (see §5).

Our first result examines the impact of certification on the cost of drug shortages to patients. Define

$$\bar{r}_H = 2r_L - r_L^2 - \frac{r_L(1-r_L)}{2}\mathbf{1}[c_L > \bar{\bar{c}}_L(r_L)], \tag{6}$$

where $\mathbf{1}[c_L > \bar{\bar{c}}_L(r_L)] = 1$ if $c_L > \bar{\bar{c}}_L(r_L)$ and 0 otherwise, and $\bar{\bar{c}}_L(r_L)$ is the same threshold defined in (5). Then we have the following:

Proposition 1 (***Impact of Certification on Patients' Shortage Cost***)**.** *(i) If $r_H < \bar{r}_H$, certification leads to a strictly higher expected shortage cost for patients than no certification.* *(ii) Otherwise, certification leads to a lower expected shortage cost for patients than no certification.*

Proposition 1 shows that certification can *increase* the shortage cost for patients even though it motivates manufacturers to improve reliability. This happens when the certification threshold for high reliability (r_H) is not high enough (below the threshold $\bar{r}_H$ defined in Equation (6)). Per Lemma 3, certification results in the buyers shifting their sourcing strategy from dual sourcing to single sourcing from a high reliability manufacturer and, potentially, reducing the sourcing quantity. If the certification threshold r_H is not sufficiently high, this shift in buyers' sourcing strategy can result in a higher risk of disruption and hence a greater shortage cost. In other words, while certification does incentivize manufacturers to improve their reliability, it can also incentivize buyers to pursue less resilient sourcing strategies (reducing the overall sourcing quantity and single, rather than dual, sourcing); if the certification threshold is not sufficiently high, the latter effect can outweigh the former, with the result that patients experience greater shortage costs with reliability certification than without.

Proposition 1 offers several insights for policymakers or nonprofit organizations developing certifications to mitigate drug shortages. In particular, it shows that certification does not always help mitigate the impact of drug shortages; in fact, in scenarios where the adoption of a higher quality production process results in only a mild reliability improvement, revealing the incremental increase in reliability can even backfire by increasing the shortage cost. Importantly, Proposition 1 also shows that when the certification threshold is sufficiently high (i.e., when $r_H \geq \bar{r}_H$), certification does help reduce the cost of shortages. This implies that policymakers should employ certification judiciously by establishing high certification standards and focusing on certifying the adoption of production processes that yield significant reliability improvements.

Next, we consider how certification affects manufacturer profit. Define

$$\bar{a} = \bar{\bar{a}} - \frac{\frac{2f}{D} + r_L \left(\bar{\bar{c}}_L(r_L) - c_L\right)^+}{r_H(r_H - r_L)}.$$

Then we have:

Proposition 2 (***Impact of Certification on Manufacturer Profit***)**.** *(i) If $a > \bar{a}$, certification leads to a strictly lower expected (average) manufacturer profit than no certification.*
(ii) Otherwise, certification leads to a higher expected manufacturer profit than no certification.

Proposition 2 shows that certification can *decrease* manufacturers' profit if the cost for reliability improvement (i.e., a) is sufficiently high. Interestingly, this result implies that certification may hurt manufacturers even when certification itself is free (i.e., $f = 0$). This happens because, with certification, in equilibrium one manufacturer improves reliability and earns a positive profit while the other does not improve reliability and earns zero profit (Lemma 3). The latter manufacturer clearly earns a lower profit with certification than without. The former manufacturer, who chooses high reliability, earns a higher profit than the no certification case (otherwise, this manufacturer would choose low reliability as in the no certification case), but this profit increase is limited if the cost for reliability improvement is sufficiently high ($a > \bar{a}$). Hence, in this case, manufacturers are on average worse off than if certification were not present.

On the other hand, Proposition 2 also shows that certification can increase the average manufacturer profit if a falls below the threshold $\bar{a}$. In this case, while it is still true that in equilibrium one manufacturer will not improve reliability and will receive zero profit, the other manufacturer improves reliability, receives all demand, and has a significantly higher profit margin than in the no certification case. Hence, on average manufacturer profit increases with certification (although one manufacturer will be a "winner" and the other will be a "loser" for the business of the buyers).

Next, we consider the impact of certification on buyers:

Proposition 3 (***Impact of Certification on Buyer Cost***)**.** *Certification leads to lower expected buyer cost than no certification.*

Proposition 3 shows that certification always benefits the buyers. Under certification, buyers only purchase from the high reliability manufacturer; intuitively, they would not make this choice unless it was less costly than purchasing from the low reliability manufacturer (even though the high reliability manufacturer charges a higher price, this price increase is limited due to price competition with the low reliability manufacturer). Moreover, in order to compete for demand, the low reliability manufacturer will further lower its price compared to the case of no certification (unless it already has zero margin). This implies that both manufacturers become (weakly) more attractive to buyers under certification compared to the case of no certification. Therefore, buyers are better off under certification.

Lastly, we consider the impact of certification on the GPO's profit. Define

$$\bar{c}_L = (2\bar{\bar{c}}_L(r_L) - (1-\eta)(\frac{r_H}{r_L} - 1)p)\mathbf{1}[c_L \leq \bar{\bar{c}}_L(r_L)].$$

Then we have:

Proposition 4 (***Impact of Certification on GPO Profit***)**.** *(i) If $c_L < \bar{c}_L$, certification leads to a strictly lower expected GPO profit than no certification.*

(ii) Otherwise, certification leads to a higher expected GPO profit than no certification.

Proposition 4 shows that certification may also reduce the GPO's profit if the production cost for the low reliability case (c_L) is sufficiently low. This happens because certification has two opposing effects on the GPO's profit. First, certification can cause buyers to decrease their total sourcing volume (as discussed following Proposition 1), which in turn reduces the quantity of purchases on which the GPO earns a commission, decreasing GPO profit. Second, certification can increase the wholesale price charged by the (high reliability) manufacturer, which increases the (per unit) revenue the GPO receives for each transaction, increasing GPO profit. Whether the net effect is an increase or decrease in GPO profit depends on the relative magnitude of these effects. If the marginal production cost under low reliability is low ($c_L < \bar{c}_L$), the increase in wholesale price charged by the high reliability manufacturer is modest due to price competition with the low reliability manufacturer, and as a result, the reduction in sourcing volume dominates the increase in wholesale price, leading to a net decrease in GPO profit. Consequently, a GPO benefits from certification only if c_L is high.

Given the differential impact of certification on the various stakeholders in the supply chain, a natural question is: can certification result in a Pareto improvement for *all* stakeholders? By comparing the conditions in the preceding four propositions, we may conclude that this can indeed occur, as the following corollary states:

Corollary 1 (***Pareto Improvement***)**.** *Certification improves all four performance measures (i.e., is Pareto improving for patients, manufacturers, buyers, and the GPO) when* $r_H \geq \bar{r}_H$, $a \leq \bar{a}$, *and* $c_L \geq \bar{c}_L$.

Corollary 1 shows that a certification system can benefit all stakeholders when (1) the certification threshold for high reliability (i.e., r_H) is sufficiently high; (2) reliability is not too costly (i.e., a is not too high);[10] and (3) the production cost before reliability improvement (i.e., c_L) is relatively high such that manufacturer profit margins are low. This result highlights that a reliability certification can generate significant societal benefits by achieving a Pareto improvement when implemented prudently—by focusing on drugs with low profit margins and available cost-effective reliability improvement options, and by establishing high certification standards. Meanwhile, our findings in this section also underscore the importance of carefully accounting for how certification may affect manufacturers' price competition and buyers' sourcing strategies to avoid unintended consequences.

5 Subsidizing Reliability Improvement

Our results thus far have shown that certification can help reduce drug shortage costs for patients and improve the performance of manufacturers, buyers, and the GPO. However, certification does not always reduce shortages and may even backfire under certain conditions. To help further mitigate drug shortages, an important additional measure considered in practice is for the government to provide subsidies or other financial incentives to drug manufacturers with high reliability (see The White House 2021a, p. 243 and Advanced Regenerative Manufacturing Institute 2022, p. 32

[10]Note that by Assumption 1, a cannot be too low either. Otherwise, if the cost of reliability improvement is very low, manufacturers would have improved their reliability even without certification.

for proposals on offering financial incentives, such as tax reductions, to drug manufacturers that adopt mature quality manufacturing process or take other measures to improve supply reliability). Such subsidies, in turn, can help to defray the cost of high reliability to manufacturers and buyers—for the latter, because subsidies prevent the cost of reliability from being passed on through the wholesale price—and might also induce more manufacturers to improve reliability, moving to a new equilibrium that results in even lower shortage costs than those found in our preceding analysis.

In this section, we analyze the impact of combining certification with a subsidy, under which certified high reliability manufacturers receive a unit subsidy $k \geq 0$. In this case, manufacturer i's expected profit is defined in a similar way as in Section 4, except that if it is certified as high reliability (i.e., $r_i \geq r_H$), it receives a subsidy k for each unit of product it sells. This effectively reduces its production cost to $c_i(r_i) - k$. For simplicity, we omit the equilibrium analysis here (details are provided in the Online Appendix). To summarize, we see that, in comparison to certification without subsidies, the addition of a subsidy can indeed motivate both manufacturers to improve reliability even with high production costs, and buyers may decide to dual source from both high reliability manufacturers.

Proposition 5 summarizes several consequences of this and other shifts in the equilibrium behavior of manufacturers and buyers on patients' shortage cost.

Proposition 5 (***Impact of Subsidies***)**.** *Compared to certification without a subsidy:*

(i) Subsidies lead to a strictly higher expected shortage cost for patients if $r_H < \bar{r}_H$*,* $a \geq \bar{\bar{a}}$ *,* $c_L > \bar{\bar{c}}_L(r_H)$*, and* $\bar{k} < k \leq \bar{\bar{k}}$*, where* $\bar{k} = (a - \bar{\bar{a}})(r_H - r_L)$ *and* $\bar{\bar{k}} = a(r_H - r_L) + c_L - \bar{\bar{c}}_L(r_H)$*.*

(ii) Otherwise, subsidies lead to a lower expected shortage cost for patients.

Proposition 5 shows that, if the subsidy level is sufficiently high (i.e., $k > \bar{\bar{k}}$), subsidies lead to a lower shortage cost than without subsidy. This is because a sufficiently high subsidy induces both manufacturers to improve reliability and induces the buyers to dual source from both manufacturers, leading to a strictly lower shortage probability and quantity. However, the proposition also shows that subsidies can backfire: this happens when the certification threshold for high reliability (r_H) is not high enough, and subsidies induce only one manufacturer to improve reliability while also resulting in buyers shifting their sourcing strategy to single source from the high reliability manufacturer. Thus, similar to the kind of backfiring that can occur from certification (Proposition 1), a subsidy that is not large enough can actually result in an *increase* in the shortage cost.

Put differently, Proposition 5 shows that providing a sufficiently high subsidy can prevent certification from backfiring and inadvertently increasing shortage costs, because it can induce *both* manufacturers to improve reliability, leading buyers to dual source. Thus, subsidizing reliability improvement can "fix" the potential negative aspects of certification. However, there is a risk: if the certification threshold is low and the subsidy amount is not high enough, it can actually *increase* the shortage cost compared to the case of no subsidies. Hence, subsidies should be used with caution: they need to be either combined with a high certification standard or set at a sufficiently high level to prevent backfiring and increased shortage costs.

6 Conclusion

Shortages of generic drugs are a persistent and costly problem in the U.S. healthcare system. Adding to that difficulty is the inherent information asymmetry about process reliability in drug manufacturing: buyers typically know little about the reliability level of generic drug manufacturers. Certification systems have been proposed by policymakers to remedy this problem by credibly verifying the adoption of high quality, high reliability manufacturing processes (FDA, 2020; The White House, 2021a). In this paper, we provide a game theoretic analysis of the impact of such certification systems on a generic drug supply chain, taking into account their effects on manufacturer reliability and pricing decisions, buyer sourcing strategies, and risk of drug shortages.

We find that even though certification does spur reliability improvement, the end result might be higher patient shortage costs if the certification threshold for high reliability is not set high enough. In addition, we also show that while certification reduces costs for buyers, it may hurt manufacturer and GPO profits. These findings suggest that universally implementing certification may be unwise; instead, certification could be targeted to certain contexts, such as by focusing on drugs with low profit margins and available cost-effective reliability improvement options, and by establishing high certification standards. Under these conditions, certification results in Pareto improvement compared to no certification by reducing patients' shortage costs while also reducing buyer costs and increasing manufacturer and GPO profits.

Finally, we find that combining certification with subsidies—where certified high reliability manufacturers are subsidized by the government—can effectively address the adverse consequences of certification and further reduce the cost of shortages, provided the subsidy is sufficiently high. This can eliminate the "backfiring" effect of certification, resolving a key issue. However, if both the subsidy and the certification threshold for high reliability are low, subsidies themselves can backfire and result in an increase in shortage costs. Thus, subsidies may need to be either combined with a high certification standard or set at a sufficiently high level to avoid potential negative outcomes.

In sum, this research represents the first theoretical investigation of the implications of a generic drug reliability certification system, which is an important shortage mitigation strategy currently under development by both policymakers and nonprofit organizations (End Drug Shortages Alliance, 2021; FDA, 2023b). Our results offer practical and timely insights regarding when and why such a certification can benefit various stakeholders in the supply chain, while cautioning against unintended consequences arising from its impact on manufacturers' price competition and buyers' sourcing strategies. Subsequent research could further build on these insights and explore additional complexities, such as the impact of heterogeneous buyers, the ramifications of inaccurate certification, and the role of asymmetric information concerning manufacturers' costs.

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Online Appendix for "Mitigating Shortages of Generic Drugs: The Role of Reliability Certification"

A Proofs of Analytical Results

A.1 Proofs of Results in Section 3

Proof of Lemma 1. Consider any buyer $j \in \{1, \ldots, N\}$. We next show that the optimal sourcing strategy satisfies (1) $Q^*_{1j} + Q^*_{2j} \geq d$ and (2) $Q^*_{ij} \leq d, i \in \{1, 2\}$. We prove these results by using contradiction:

- If $Q_{1j} + Q_{2j} < d$, when we increase Q_{1j} by a very small quantity ϵ (i.e., $Q_{1j} + Q_{2j} + \epsilon \leq d$), the buyer's expected cost (Equation 2) would decrease by $\mathbb{E}_j[r_1(p - w_1)\epsilon] > 0$. Thus, we have $Q^*_{1j} + Q^*_{2j} \geq d$.
- If $Q_{ij} > d$ for some $i \in \{1, 2\}$, when we decrease Q_{ij} by a very small quantity ϵ (i.e., $Q_{ij} - \epsilon \geq d$), the buyer's expected cost would decrease by $\mathbb{E}_j[r_i(w_i - s)\epsilon] > 0$. Thus, we have $Q^*_{ij} \leq d$.

Having proved that $Q^*_{1j} + Q^*_{2j} \geq d$ and $Q^*_{ij} \leq d$, $i \in \{1, 2\}$, it is sufficient to focus on Q_{1j} and Q_{2j} such that $Q_{1j} + Q_{2j} \geq d$ and $Q_{ij} \leq d$. This allows us to simplify buyer j's cost in Equation 2 by replacing $(d - Q_{1j} - Q_{2j})^+$, $(Q_{1j} - d)^+$, $(Q_{2j} - d)^+$ by 0. Further, $Q_{1j} + Q_{2j} - d$, $d - Q_{1j}$, and $d - Q_{2j}$ are all non-negative. As a result, buyer j's objective can be simplified as follows:

$$\sum_{i=1}^{2} \mathbb{E}_j \left[((1 - r_1 r_2)p + r_1 r_2 s)\, d - \sum_{i=1}^{2} r_i \left((1 - r_{-i})\, p + r_{-i} s - w_i \right) Q_{ij} \right]$$
$$= \underbrace{\sum_{i=1}^{2} \mathbb{E}_j \left[((1 - r_1 r_2)p + r_1 r_2 s)\, d \right]}_{\text{Constant}} - \sum_{i=1}^{2} \mathbb{E}_j[r_i \left((1 - r_{-i})\, p + r_{-i} s - w_i \right)] Q_{ij}.$$

We can see that buyer j's optimal sourcing strategy is determined by $\mathbb{E}_j[r_i \left((1 - r_{-i})\, p + r_{-i} s - w_i \right)]$ for $i \in \{1, 2\}$. We next analyze buyer j's optimal strategy in three scenarios:

(i) If $\mathbb{E}_j[r_i \left((1 - r_{-i})\, p + r_{-i} s - w_i \right)] \geq 0, i \in \{1, 2\}$, sourcing more from either manufacturer will decrease the buyer's cost. Therefore, the buyer will source d units from each manufacturer.

(ii) If $\mathbb{E}_j[r_i \left((1 - r_{-i})\, p + r_{-i} s - w_i \right)] \geq 0$ while $\mathbb{E}_j[r_{-i} \left((1 - r_i)\, p + r_i s - w_{-i} \right)] < 0$ for $i \in \{1, 2\}$, sourcing more from manufacturer i will decrease the buyer's cost while sourcing more from manufacturer $-i$ will increase the cost. Therefore, each buyer will source d from manufacturer i and nothing from $-i$.

(iii) If $\mathbb{E}_j[r_i \left((1 - r_{-i})\, p + r_{-i} s - w_i \right)] < 0, i \in \{1, 2\}$, sourcing more from either manufacturer will increase the buyer's cost. Thus, the total sourcing quantity of the buyer must be d in this case, and the buyer will select the manufacturer(s) to minimize cost. Specifically,

- if sourcing from manufacturer i leads to a strictly lower cost than sourcing from manufacturer $-i$, i.e., $\mathbb{E}_j[r_i((1-r_{-i})p + r_{-i}s - w_i)] > \mathbb{E}_j[r_{-i}((1-r_i)p + r_i s - w_{-i})]$, the buyer will source d from manufacturer i and nothing from manufacturer $-i$;
- if sourcing from manufacturer i leads to the same cost with sourcing from manufacturer $-i$, i.e., $\mathbb{E}_j[r_i((1-r_{-i})p + r_{-i}s - w_i)] = \mathbb{E}_j[r_{-i}((1-r_i)p + r_i s - w_{-i})]$, the buyer will source αd from manufacturer i and $(1-\alpha)d$ from manufacturer $-i$, where $\alpha \in [0,1]$. $\square$

Proof of Lemma 2. As each buyer's sourcing decisions have been characterized in Lemma 1, it remains to identify each manufacturer's reliability and pricing decisions in equilibrium.

By Assumption 1, we have $a \geq (1-\eta)\frac{(1-r_L^2)p + r_L^2 s}{r_L} \geq (1-\eta)\frac{\left((1-r_L)p + r_L s - \frac{c_L}{1-\eta}\right)^+}{r_L}$. From each manufacturer's objective in Equation 1, we can see that the minimum wholesale price that allows for a non-negative profit is $\frac{c_L}{1-\eta}$. Recall from Equation 1 that manufacturer i's expected profit under no certification is given by

$$\begin{aligned}\Pi_i^{\text{Non-certified}}(r_i, w_i|w_{-i}) &= \sum_{j=1}^{N} r_i\left((1-\eta)w_i - c_i(r_i)\right)Q_{ij}(w_1, w_2)\\ &= \sum_{j=1}^{N}\left(-ar_i^2 + ((1-\eta)w_i - c_L + ar_L)\, r_i\right)Q_{ij}(w_1, w_2).\end{aligned}$$

Clearly, manufacturer i's expected profit increases with r_i when $r_i < \frac{(1-\eta)w_i - c_L}{2a} + \frac{r_L}{2}$ and decreases with r_i when $r_i > \frac{(1-\eta)w_i - c_L}{2a} + \frac{r_L}{2}$. By the definition of PBE, all players' decisions must be sequentially rational. Then, since $r_L \leq r_i \leq 1$, in equilibrium, the reliability level of manufacturer i must satisfy

$$r_i(w_i) = \min\{1, \max\{r_L, \frac{(1-\eta)w_i - c_L}{2a} + \frac{r_L}{2}\}\}, \quad \text{where } \frac{c_L}{1-\eta} \leq w_i < p, \tag{A.1}$$

otherwise, the manufacturer would have an incentive to deviate to a different reliability level. This also implies that after observing the wholesale price w_i of each manufacturer, each buyer's belief on the manufacturer's reliability choice r_i must be consistent with Equation A.1. Note that we use the notation $r_i(w_i)$ to represent the relationship between r_i and w_i in equilibrium, rather than to suggest that r_i is chosen in response to w_i.

(i) Suppose $\frac{c_L}{1-\eta} \leq (1-r_L)p + r_L s$. In this case, we show that we must have $r_1 = r_2 = r_L$ and $w_1 = w_2 = (1-r_L)p + r_L s$ in equilibrium. We prove this in the following two steps:

First, we show that the strategy profile where $r_1 = r_2 = r_L$ and $w_1 = w_2 = (1-r_L)p + r_L s$, together with buyers' sourcing strategies characterized in Lemma 1 and beliefs that are consistent with manufacturers' reliability choices, constitutes a PBE. Given buyers' sourcing strategies characterized in Lemma 1, it is sufficient to prove that neither manufacturer has the incentive to deviate.

For $i \in \{1,2\}$, by equation A.1 and Assumption 1 (which implies $a \geq (1-\eta)\frac{\left((1-r_L)p + r_L s - \frac{c_L}{1-\eta}\right)^+}{r_L}$), given $w_i = (1-r_L)p + r_L s$ and $\frac{c_L}{1-\eta} \leq (1-r_L)p + r_L s$, we have $r_i(w_i) = r_L$. Therefore, it is sufficient to prove that manufacturer i will not have the incentive to deviate to a different wholesale price w_i, where its reliability level is given by $r_i = r_i(w_i)$ as characterized in Equation A.1.

Given $r_1 = r_2 = r_L$ and $w_1 = w_2 = (1-r_L)p + r_L s$, together with beliefs that are consistent with manufacturers' reliability choices, we know from Lemma 1 that each manufacturer receives a demand quantity d from each buyer (see the first scenario in Lemma 1).

(a) If manufacturer 1 deviates to a lower price (i.e., $w_1 < (1-r_L)p+r_Ls$) and chooses $r_1 = r_1(w_1) = r_L$ (per Equation A.1), it will still receive d but at a reduced price, resulting in a lower profit.

(b) If manufacturer 1 deviates to a higher price that is lower than $\frac{ar_L+c_L}{1-\eta}$ and chooses $r_1 = r_1(w_1) = r_L$, we have $w_1 > (1-r_L)p + r_Ls \geq w_2$. Then, manufacturer 1 will receive nothing from buyers (see the second scenario in Lemma 1). Therefore, deviating to a higher price that is lower than $\frac{ar_L+c_L}{1-\eta}$ leads to a zero profit.

(c) If manufacturer 1 deviates to a higher price that is strictly higher than $\frac{ar_L+c_L}{1-\eta}$ and chooses $r_1 = r_1(w_1) > r_L$, we now prove that it will receive nothing from buyers. First, based on Assumption 1 (i.e., $a \geq (1-\eta)\frac{(1-r_L^2)p+r_L^2s}{r_L}$), we have $\frac{ar_L+c_L}{1-\eta} \geq (1-r_L^2)p + r_L^2s + \frac{c_L}{1-\eta} > (1-\frac{r_L^2}{r_1})p + \frac{r_L^2}{r_1}s$. Then, based on the third scenario in Lemma 1, we can prove that manufacturer 1 will receive nothing from buyers if $w_1 > (1-\frac{r_L^2}{r_1})p + \frac{r_L^2}{r_1}s$, $r_1 = r_1(w_1)$, $w_2 = (1-r_L)p + r_Ls$, and $r_2 = r_L$, as follows: $w_1 > (1-\frac{r_L^2}{r_1})p+\frac{r_L^2}{r_1}s = (1-\frac{r_L}{r_1})p+\frac{r_L}{r_1}w_2 \iff r_L((1-r_1)p+r_1s-w_2) > r_1((1-r_L)p+r_Ls-w_1)$. Therefore, deviating to $w_1 > \frac{ar_L+c_L}{1-\eta}$ leads to a zero profit of manufacturer 1.

Combining all cases, we conclude that the strategy profile with $r_1 = r_2 = r_L$ and $w_1 = w_2 = (1-r_L)p + r_Ls$ constitutes an equilibrium. Moreover, based on Lemma 1, each buyer sources d from each manufacturer in equilibrium.

Second, we show that any other strategy cannot be an equilibrium. We prove this by showing that at least one manufacturer has an incentive to deviate in any other scenario.

(a) If only one manufacturer, say, manufacturer 2, sets its wholesale price at $(1-r_L)p + r_Ls$, i.e., $w_2 = (1-r_L)p+r_Ls \neq w_1$, then manufacturer 1 has an incentive to deviate to $w_1 = (1-r_L)p+r_Ls$ according to the analysis above.

(b) If $w_1, w_2 < (1-r_L)p + r_Ls$ (which implies $r_1(w_1) = r_2(w_2) = r_L$ by Equation A.1 and Assumption 1), then we have $w_i \leq (1-r_{-i}(w_{-i}))p+r_{-i}(w_{-i})s, i \in \{1,2\}$. In this case, each buyer sources d from each manufacturer. Then, each manufacturer will have an incentive to increase the price to $(1-r_L)p + r_Ls$ so that it still obtains a demand d from each buyer but at a higher price.

(c) If $w_1, w_2 > (1-r_L)p + r_Ls$, then we have $w_i > (1-r_{-i}(w_{-i}))p + r_{-i}(w_{-i})s, i \in \{1,2\}$. In this case, each buyer will source d in total (see the third scenario in Lemma 1). Thus, at least one of the manufacturers receives 0 or $\frac{1}{2}d$ from each buyer. We next show that the manufacturer who receives 0 or $\frac{1}{2}d$ from each buyer will have an incentive to deviate to a lower price.

- If one of the manufacturers, say, manufacturer 1, receives nothing from each buyer, then it can benefit from deviating to $w_1 = w_2$ (which implies $r_1(w_1) = r_2(w_2)$ by Equation A.1) such that it receive $\frac{d}{2}$ from each buyer, resulting in a strictly positive profit.

- If each manufacturer receives $\frac{1}{2}d$ from each buyer, which happens when sourcing from either manufacturer leads to the same cost for buyers, i.e., $r_1(w_1)((1-r_2(w_2))p + r_2(w_2)s - w_1) = r_2(w_2)((1-r_1(w_1))p + r_1(w_1)s - w_2)$, we next show that one of the manufacturers has an incentive to deviate. We divide the analysis into the following three cases:

  (1) When $w_i > \frac{a(2-r_L)+c_L}{1-\eta}$ for $i=1$ or $i=2$, by Equation A.1, we have $r_i(w_i) = 1$. Then, based on the third scenario of Lemma 1, by deviating to a slightly lower price (such that we still have $w_i > \max\{\frac{a(2-r_L)+c_L}{1-\eta}, (1-r_L)p + r_L s\}$ and $r_i(w_i) = 1$), manufacturer i can receive a demand d from each buyer, resulting in a strictly higher profit.

  (2) When $w_i \leq \frac{ar_L+c_L}{1-\eta}$ for $i=1$ or $i=2$, by Equation A.1, we have $r_i(w_i) = r_L$. Then, based on the third scenario of Lemma 1, by deviating to a slightly lower price (such that w_i is still higher than $(1-r_L)p + r_L s$ and $r_i(w_i)$ remains unchanged), manufacturer i can receive a demand d from each buyer, resulting in a strictly higher profit.

  (3) Otherwise (i.e., $\frac{ar_L+c_L}{1-\eta} < w_i \leq \frac{a(2-r_L)+c_L}{1-\eta}$ for $i \in \{1,2\}$), by Equation A.1, we have $r_i(w_i) = \frac{(1-\eta)w_i - c_L}{2a} + \frac{r_L}{2} \in (r_L, 1]$ for $i \in \{1,2\}$. We next prove that by deviating to a slightly different price, manufacturer i can receive a demand d from each buyer, resulting in a strictly higher profit. Let $r_1 = r_1(w_1)$ and $r_2 = r_2(w_2)$. Suppose manufacturer 1 deviates from w_1 to $w_1 - \Delta$ such that $\max\{\frac{ar_L+c_L}{1-\eta}, (1-r_L)p + r_L s\} < w_1 - \Delta \leq \frac{a(2-r_L)+c_L}{1-\eta}$, where Δ could be either positive (i.e., deviating to a lower price) or negative (i.e., deviating to a higher price). Then, by Equation A.1, we have $r_1(w_1 - \Delta) = \frac{(1-\eta)(w_1-\Delta)-c_L}{2a} + \frac{r_L}{2} > \frac{(1-\eta)\frac{ar_L+c_L}{1-\eta} - c_L}{2a} + \frac{r_L}{2} = r_L$. We also have $r_1(w_1 - \Delta) = r_1 - \frac{1-\eta}{2a}\Delta$. Then, based on the third scenario of Lemma 1, manufacturer 1 will receive a demand d from each buyer after deviating if $(r_1 - \frac{1-\eta}{2a}\Delta)((1-r_2)p + r_2 s - w_1 + \Delta) > r_2((1 - r_1 + \frac{1-\eta}{2a}\Delta)p + (r_1 - \frac{1-\eta}{2a}\Delta)s - w_2)$. Since $r_1((1-r_2)p + r_2 s - w_1) = r_2((1-r_1)p + r_1 s - w_2)$, the above inequality holds for some Δ such that $\Delta(-\Delta + \frac{2ar_1}{1-\eta} - p + w_1) > 0$. Therefore, when $w_1 < p - \frac{2ar_1}{1-\eta}$, manufacturer 1 has the incentive to deviate to a slightly higher price (i.e., Δ is negative), and it has the incentive to deviate to a slightly lower price (i.e., Δ is positive) otherwise. And following Assumption 1 (i.e., $a \geq \frac{(1-\eta)p - c_L}{4-r_L}$), we have $\frac{a(2-r_L)+c_L}{1-\eta} \geq p - \frac{2a}{1-\eta}$. Thus, manufacturer 1 has the incentive to deviate to a slightly lower price if $w_1 = \frac{a(2-r_L)+c_L}{1-\eta}$ and $r_1 = 1$.

(d) If one manufacturer, say, manufacturer 1, sets its wholesale price to be strictly higher than $(1-r_L)p + r_L s$ but manufacturer 2's wholesale price is strictly lower than $(1-r_L)p + r_L s$ (i.e., $w_1 > (1-r_L)p + r_L s > w_2$), then following a similar analysis as for proving that both manufacturers setting a price $(1-r_L)p + r_L s$ is an equilibrium, we have that manufacturer 1 receives nothing from the buyers. In this case, manufacturer 1 can gain a demand d from each buyer and a strictly positive profit by lowering its price to $w_1 = (1-r_L)p + r_L s$.

(ii) Suppose $\frac{c_L}{1-\eta} > (1-r_L)p + r_L s$. In this case, we show that we must have $r_1 = r_2 = r_L$ and $w_1 = w_2 = \frac{c_L}{1-\eta}$ in equilibrium. We prove this in the following two steps:

First, we show that the strategy profile where $r_1 = r_2 = r_L$ and $w_1 = w_2 = \frac{c_L}{1-\eta}$ together with buyers' sourcing strategies characterized in Lemma 1 and beliefs that are consistent with manufacturers' reliability choices, constitutes a PBE. Given buyers' sourcing strategies characterized in Lemma 1, it is sufficient to prove that neither manufacturer has the incentive to deviate.

For $i \in \{1, 2\}$, by equation A.1, given $w_i = \frac{c_L}{1-\eta}$, we have $r_i(w_i) = r_L$. Therefore, it is sufficient to prove that manufacturer i will not have the incentive to deviate to a different wholesale price w_i, where its reliability level is given by $r_i = r_i(w_i)$ as characterized in Equation A.1.

(a) If manufacturer 1 deviates to a wholesale price that satisfies $\frac{c_L}{1-\eta} < w_1 \leq \frac{a r_L + c_L}{1-\eta}$ and chooses $r_1 = r_1(w_1) = r_L$ (per Equation A.1), it will receive nothing from buyers (see the third scenario in Lemma 1), resulting in a zero profit for this manufacturer.

(b) If manufacturer 1 deviates to a wholesale price that satisfies $w_1 > \frac{a r_L + c_L}{1-\eta}$ and chooses $r_1 = r_1(w_1) > r_L$, we now prove that it will receive nothing from buyers. Specifically, by Assumption 1 (i.e., $a \geq (1-\eta)\frac{(1-r_L^2)p + r_L^2 s}{r_L}$), we have $w_1 > \frac{a r_L + c_L}{1-\eta} \implies w_1 > (1-r_L^2)p + r_L^2 s + \frac{c_L}{1-\eta} > (1-r_L)p + r_L s + \frac{c_L}{1-\eta}$. Then, based on the third scenario in Lemma 1, we can prove that manufacturer 1 will receive nothing from buyers if $w_1 > (1-r_L)p + r_L s + \frac{c_L}{1-\eta}$, $r_1 = r_1(w_1)$, $w_2 = \frac{c_L}{1-\eta}$, and $r_2 = r_L$, as follows: $w_1 > (1-r_L)p + r_L s + \frac{c_L}{1-\eta} > (1 - \frac{r_L}{r_1})p + \frac{r_L}{r_1} w_2 \implies r_1((1-r_L)p + r_L s - w_1) < r_L((1-r_1)p + r_1 s - w_2)$. Therefore, deviating to $w_1 > \frac{a r_L + c_L}{1-\eta}$ leads to a zero profit of manufacturer 1.

Combining both cases, we conclude that the strategy profile with $r_1 = r_2 = r_L$ and $w_1 = w_2 = \frac{c_L}{1-\eta}$ constitutes an equilibrium. Moreover, by Lemma 1, each buyer sources $\frac{1}{2}d$ from each manufacturer.

Second, we show that any other strategy cannot be an equilibrium. We prove this by showing that at least one manufacturer has an incentive to deviate in any other scenario.

(a) If only one manufacturer sets its wholesale price at $\frac{c_L}{1-\eta}$, say, $w_2 = \frac{c_L}{1-\eta} < w_1$, then manufacturer 1 has an incentive to deviate to $w_1 = \frac{c_L}{1-\eta}$ according to the analysis above (note that by the tie-breaking rule, each manufacturer chooses the lowest price that results in the same profit).

(b) If $w_1, w_2 > \frac{c_L}{1-\eta}$, based on the third scenario of Lemma 1, each buyer sources d in total and at least one of the manufacturer receives 0 or $\frac{1}{2}d$. Following a similar analysis as in case (c) of the second step of case (i), the manufacturer who receives 0 or $\frac{1}{2}d$ from each buyer will have an incentive to deviate.

Combining our analysis for both cases, our conclusion in Lemma 2 holds. □

A.2 Proofs of Results in Section 4

Proof of Lemma 3. We prove this lemma in the following three steps.

Step 1: We first analyze buyers' sourcing strategies. With certification, buyers observe (1) the certification status δ_i of each manufacturer (i.e., whether the manufacturer's reliability is higher than or equal to r_H), and (2) the wholesale price w_i of each manufacturer. Therefore, our characterization of buyers' sourcing strategies in Lemma 1 continues to hold, except that buyers' beliefs about each manufacturer's reliability choice now depend on both the certification statuses and wholesale prices of the two manufacturers.

Step 2: For each certification status profile (δ_1, δ_2) that may arise in equilibrium, we derive necessary conditions that the corresponding equilibrium reliability levels and prices must satisfy. Let c_H denote the production cost if a manufacturer chooses a reliability level of r_H, i.e., $c_H = c_L + a(r_H - r_L)$. Then, to maintain a non-negative profit, it is sufficient to consider a wholesale price $w_i \geq \frac{c_H}{1-\eta}$ for a certified high reliability manufacturer. Following a similar analysis as in Lemma 2, in equilibrium, each manufacturer i's reliability level r_i, certification status δ_i, and wholesale price w_i must satisfy the following:

$$r_i(\delta_i, w_i) = \begin{cases} \min\{r_H - \epsilon, \max\{r_L, \frac{(1-\eta)w_i - c_L}{2a} + \frac{r_L}{2}\}\} & \text{if } \delta_i = 0 \text{ and } \frac{c_L}{1-\eta} \leq w_i < p \\ \min\{1, \max\{r_H, \frac{(1-\eta)w_i - c_L}{2a} + \frac{r_L}{2}\}\} & \text{if } \delta_i = 1 \text{ and } \frac{c_H}{1-\eta} \leq w_i < p, \end{cases} \tag{A.2}$$

otherwise, the manufacturer would have an incentive to deviate to a different reliability level. This also implies that after observing the certification status δ_i and wholesale price w_i of each manufacturer, each buyer's belief on the manufacturer's reliability choice r_i must be consistent with Equation A.2. Note that we use the notation $r_i(\delta_i, w_i)$ to represent the relationship between r_i, δ_i, and w_i in equilibrium, rather than to suggest that r_i is chosen in response to δ_i and w_i.

(i) Suppose we have $\delta_1 = \delta_2 = 0$ in equilibrium (i.e., neither manufacturer is certified as high reliability). Following a similar analysis as in the proof of Lemma 2, we must have $r_1 = r_2 = r_L$ and $w_1 = w_2 = \max\{\frac{c_L}{1-\eta}, (1 - r_L)p + r_L s\}$ in equilibrium.

(ii) Suppose we have $\delta_1 = \delta_2 = 1$ in equilibrium (i.e., both manufacturers are certified as high reliability). Following a similar analysis as in the proof of Lemma 2, we must have $r_1 = r_2 = r_H$ and $w_1 = w_2 = \max\{\frac{c_H}{1-\eta}, (1 - r_H)p + r_H s\}$ in equilibrium.

(iii) Suppose we have $\delta_1 = 1, \delta_2 = 0$ in equilibrium (i.e., manufacturer 1 is certified as high reliability while manufacturer 2 is not). Then, it is without loss of generality to consider $w_1 \geq \frac{c_H}{1-\eta}$ and $w_2 \geq \frac{c_L}{1-\eta}$.

   (a) When $\frac{c_H}{1-\eta} \leq (1 - r_L)p + r_L s$ and $\frac{c_L}{1-\eta} \leq (1 - r_H)p + r_H s$, following a similar analysis as in the proof of Lemma 2, we must have $r_1 = r_H, r_2 = r_L$ and $w_1 = (1-r_L)p+r_L s$, $w_2 = (1-r_H)p+r_H s$ in equilibrium.

   (b) When $\frac{c_H}{1-\eta} > (1-r_L)p+r_L s$ and $\frac{c_L}{1-\eta} \leq (1-r_H)p+r_H s$, implying $\frac{r_H c_H - r_L c_L}{1-\eta} > (r_H - r_L)p$, we next show that there must be no equilibrium that satisfies $\delta_1 = 1, \delta_2 = 0$. Given $\frac{c_H}{1-\eta} > (1-r_L)p+r_L s$, for any $w_1 \in [\frac{c_H}{1-\eta}, p)$ and $w_2 \in [\frac{c_L}{1-\eta}, p)$, we have $w_1 > (1 - r_2(0, w_2))\, p + r_2(0, w_2)s$. Moreover, by simple algebra, if $r_1(1, w_1)((1 - r_2(0, w_2))p + r_2(0, w_2)s - w_1) \geq r_2(0, w_2)((1 - r_1(1, w_1))p + r_1(1, w_1)s - w_2)$, then we have $w_2 > (1 - r_1(1, w_1))\, p + r_1(1, w_1)s$. Consider the following three sub-cases:

- If $r_1(1, w_1)((1 - r_2(0, w_2))p + r_2(0, w_2)s - w_1) = r_2(0, w_2)((1 - r_1(1, w_1))p + r_1(1, w_1)s - w_2)$, based on the third scenario of Lemma 1, each buyer will source $\frac{d}{2}$ from each manufacturer. Then, following a similar analysis as in case (c) of the second step of case (i) in the proof of Lemma 2, at least one of the manufacturers has the incentive to deviate to a different price. Thus, there must be no equilibrium that satisfies that condition of this sub-case.
- If $r_1(1, w_1)((1 - r_2(0, w_2))p + r_2(0, w_2)s - w_1) > r_2(0, w_2)((1 - r_1(1, w_1))p + r_1(1, w_1)s - w_2)$, based on the third scenario of Lemma 1, each buyer will source d from manufacturer 1 and nothing from manufacturer 2. This cannot be an equilibrium because manufacturer 2 can benefit from lowering its price to $w_2 \in (\frac{c_L}{1-\eta}, (1 - r_1(1, w_1))\, p + r_1(1, w_1)s]$ (note that $(1 - r_1(1, w_1))p + r_1(1, w_1)s \geq (1 - r_H)p + r_H s \geq \frac{c_L}{1-\eta}$), so that it receives d from each buyer, resulting in a strictly positive profit (see the first and second scenarios of Lemma 1).
- If $r_1(1, w_1)((1 - r_2(0, w_2))p + r_2(0, w_2)s - w_1) < r_2(0, w_2)((1 - r_1(1, w_1))p + r_1(1, w_1)s - w_2)$, based on the second and third scenarios of Lemma 1, each buyer will source nothing from manufacturer 1 and d from manufacturer 2. In this case, manufacturer 1 has zero profit. By the tie-breaking rule, manufacturer 1 will deviate by choosing a lower reliability level. Therefore, there must be no equilibrium that satisfies that condition of this sub-case.

(c) When $\frac{c_H}{1-\eta} \leq (1-r_L)p+r_L s$ and $\frac{c_L}{1-\eta} > (1-r_H)p+r_H s$, implying $\frac{r_H c_H - r_L c_L}{1-\eta} < (r_H - r_L)p$, we next show that we must have $r_1 = r_H, r_2 = r_L$ and $w_1 = \frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} - \epsilon, w_2 = \frac{c_L}{1-\eta}$ in equilibrium. Following a similar analysis for case (b), we know that there must be no equilibrium that satisfies $r_1(1, w_1)((1-r_2(0, w_2))p+r_2(0, w_2)s-w_1) \leq r_2(0, w_2)((1-r_1(1, w_1))p+r_1(1, w_1)s-w_2)$. Thus, it is sufficient to consider $r_1(1, w_1)((1 - r_2(0, w_2))p + r_2(0, w_2)s - w_1) > r_2(0, w_2)((1 - r_1(1, w_1))p + r_1(1, w_1)s - w_2)$. Given $\frac{c_L}{1-\eta} > (1 - r_H)p + r_H s$, we have $w_2 \geq \frac{c_L}{1-\eta} > (1 - r_1(1, w_1))p + r_1(1, w_1)s$. Then, based on the second and third scenarios of Lemma 1, each buyer will source d from manufacturer 1 and nothing from manufacturer 2. This can be an equilibrium only if $w_2 = \frac{c_L}{1-\eta}$ and $r_2 = r_2(0, w_2) = r_L$ (otherwise, manufacturer 2 can lower its price to gain a weakly higher profit). Since $r_1(1, w_1)((1 - r_L)p + r_L s - w_1) > r_L((1 - r_1(1, w_1))\, p + r_1(1, w_1)s - \frac{c_L}{1-\eta})$ implies $w_1 < \frac{r_L \frac{c_L}{1-\eta} + (r_1(1,w_1) - r_L)p}{r_1(1,w_1)}$, we next prove that we must have $w_1 = \frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} - \epsilon$ and $r_1 = r_1(1, w_1) = r_H$ in equilibrium. Consider the following two sub-cases:

- If $w_1 \leq \frac{a(2r_H - r_L) + c_L}{1-\eta}$, based on Equation A.2, we have $r_1(1, w_1) = r_H$. Further, under Assumption 1, it is straightforward to check that $\frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} < \frac{a(2r_H - r_L) + c_L}{1-\eta}$. Therefore, based on the analysis above, we conclude that among all $w_1 \leq \frac{a(2r_H - r_L) + c_L}{1-\eta}$, the only value of w_1 that may support an equilibrium is $w_1 = \frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} - \epsilon$. (Note that given

$\frac{r_H c_H - r_L c_L}{1-\eta} < (r_H - r_L)p$, we have $\frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} > \frac{c_H}{1-\eta}$.)

- If $w_1 > \frac{a(2r_H - r_L) + c_L}{1-\eta}$, based on Equation A.2, we have $r_1(1, w_1) > r_H$. We now prove that given $w_1 > \frac{a(2r_H - r_L) + c_L}{1-\eta}, w_2 = \frac{c_L}{1-\eta}$ and $r_1 = r_1(1, w_1), r_2 = r_2(0, w_2)$, based on the third scenario of Lemma 1, manufacturer 1 will receive nothing from buyers. Specifically, given $w_1 > \frac{a(2r_H - r_L) + c_L}{1-\eta} > \frac{a r_L + c_L}{1-\eta}$, by Assumption 1 (i.e., $a \geq (1-\eta)\frac{(1-r_L^2)p + r_L^2 s}{r_L}$), we have $w_1 > (1 - r_L)p + r_L s + \frac{c_L}{1-\eta}$. Then, we have $w_1 > (1 - \frac{r_L}{r_1(1,w_1)})p + \frac{r_L}{r_1(1,w_1)} \frac{c_L}{1-\eta} \implies r_1(1, w_1)((1 - r_L)p + r_L s - w_1) < r_L((1 - r_1(1, w_1))p + r_1(1, w_1)s - \frac{c_L}{1-\eta})$. Therefore, there must be no equilibrium that satisfies $w_1 > \frac{a(2r_H - r_L) + c_L}{1-\eta}$.

Therefore, under the conditions of case (c), we conclude that if we have $\delta_1 = 1, \delta_2 = 0$ in equilibrium, we must have $r_1 = r_H, r_2 = r_L$ and $w_1 = \frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} - \epsilon, w_2 = \frac{c_L}{1-\eta}$ in equilibrium.

(d) When $\frac{c_H}{1-\eta} > (1 - r_L)p + r_L s$ and $\frac{c_L}{1-\eta} > (1 - r_H)p + r_H s$, following a similar analysis for case (b) and case (c), we must have $r_1 = r_H, r_2 = r_L$ and $w_1 = \frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} - \epsilon, w_2 = \frac{c_L}{1-\eta}$ in equilibrium when $\frac{r_H c_H - r_L c_L}{1-\eta} < (r_H - r_L)p$, and there must be no equilibrium otherwise.

(iv) Suppose we have $\delta_1 = 0, \delta_2 = 1$ in equilibrium (i.e., manufacturer 2 is certified as high reliability while manufacturer 1 is not). The equilibrium is symmetric to the case where $\delta_1 = 1, \delta_2 = 0$.

Step 3: Finally, we characterize the conditions under which each certification status profile arises in equilibrium. Since we have already derived the necessary conditions in Step 2 for each certification status profile to be supported in equilibrium, to check whether a certification status profile can sustain as an equilibrium, it remains to check whether any manufacturer has an incentive to deviate. Let $\Pi_i(\delta_1, \delta_2)$ denote the manufacturer i's expected profit given the certification status profile (δ_1, δ_2), where the corresponding reliability and pricing choices satisfy the equilibrium conditions characterized in Step 2. For each given certification status profile, we now examine whether either manufacturer has an incentive to deviate. Due to symmetry, it is sufficient to examine whether manufacturer 1 has an incentive to deviate. To do so, we next compare $\Pi_1(0, 0)$ with $\Pi_1(1, 0)$, and compare $\Pi_1(0, 1)$ with $\Pi_1(1, 1)$. If the manufacturer's expected profit under a certification status profile is dominated, this profile cannot be sustained as an equilibrium.

Recall that $c_H = c_L + a(r_H - r_L)$. Let

$$\bar{\bar{c}}_H = c_L + \bar{\bar{a}}(r_H - r_L) = \min\{\frac{2r_L c_L + (1-\eta)\left((r_L^2 - 2r_L + r_H)p - r_L^2 s\right)}{r_H}, \frac{r_L c_L + (1-\eta)(r_H - r_L)p}{r_H}\}.$$

Then, we have $c_H < \bar{\bar{c}}_H \iff a < \bar{\bar{a}}$. Following Assumption 1, we also have that

$$a \geq (1-\eta)\frac{(1-r_L^2)p + r_L^2 s}{r_L} > (1-\eta)\frac{(1-r_L)p + r_L s - \frac{c_L}{1-\eta}}{r_H}$$
$$\implies \frac{r_H c_H - r_L c_L}{(1-\eta)(r_H - r_L)} > (1 - r_L)p + r_L s > (1 - r_H)p + r_H s.$$

(i) We first compare $\Pi_1(0, 0)$ with $\Pi_1(1, 0)$. Consider the following three cases:

(a) Suppose $\frac{c_H}{1-\eta} \leq (1 - r_L)p + r_L s$, $\frac{c_L}{1-\eta} \leq (1 - r_H)p + r_H s$. From our analysis in Step 2, we know that if we have $(\delta_1, \delta_2) = (0, 0)$ in equilibrium, then in equilibrium, we must have $r_1 = r_L$

and manufacturer 1 will receive d from each buyer at the price of $\max\{\frac{c_L}{1-\eta}, (1-r_L)p + r_L s\} = (1-r_L)p + r_L s$. Therefore, we have $\Pi_1(0,0) = (1-\eta)r_L((1-r_L)p + r_L s - \frac{c_L}{1-\eta})D$. On the other hand, if we have $(\delta_1, \delta_2) = (1,0)$ in equilibrium, then in equilibrium, we must have $r_1 = r_H$ and manufacturer 1 will receive d from each buyer at the price of $(1-r_L)p + r_L s$. Therefore, we have $\Pi_1(1,0) = (1-\eta)r_H((1-r_L)p + r_L s - \frac{c_H}{1-\eta})D$. Then, we have

$$\begin{aligned}\Pi_1(0,0) - \Pi_1(1,0) &= (1-\eta)r_L\left((1-r_L)p + r_L s - \frac{c_L}{1-\eta}\right)D - (1-\eta)r_H\left((1-r_L)p + r_L s - \frac{c_H}{1-\eta}\right)D \\ &= (1-\eta)\left(\frac{r_H c_H - r_L c_L}{1-\eta} - (r_H - r_L)\left((1-r_L)p + r_L s\right)\right)D > 0,\end{aligned}$$

where the inequality follows from $\frac{r_H c_H - r_L c_L}{(1-\eta)(r_H - r_L)} > (1-r_L)p + r_L s$.

Therefore, in this case, $(\delta_1, \delta_2) = (0,0)$ remains a possible certification status profile that can be sustained in equilibrium, while there must be no equilibrium that satisfies $(\delta_1, \delta_2) = (1,0)$.

(b) Suppose (1) $\frac{c_H}{1-\eta} > (1-r_L)p + r_L s$ or $\frac{c_L}{1-\eta} \leq (1-r_H)p + r_H s$, and (2) $\frac{r_H c_H - r_L c_L}{1-\eta} \geq (r_H - r_L)p$. Based on our analysis in Step 2, there must be no equilibrium that satisfies $(\delta_1, \delta_2) = (1,0)$, while $(\delta_1, \delta_2) = (0,0)$ remains a possible certification status profile that can be sustained in equilibrium.

(c) Suppose (1) $\frac{c_H}{1-\eta} > (1-r_L)p + r_L s$ or $\frac{c_L}{1-\eta} > (1-r_H)p + r_H s$, and (2) $\frac{r_H c_H - r_L c_L}{1-\eta} < (r_H - r_L)p$. From our analysis in Step 2, we know that if we have $(\delta_1, \delta_2) = (0,0)$ in equilibrium, then in equilibrium, we must have $r_1 = r_L$ and manufacturer 1 will receive d from each buyer at the price of $(1-r_L)p + r_L s$ when $\frac{c_L}{1-\eta} \leq (1-r_L)p + r_L s$, in which case $\Pi_1(0,0) = (1-\eta)r_L\big((1-r_L)p + r_L s - \frac{c_L}{1-\eta}\big)D$, and it will receive $\frac{1}{2}d$ from each buyer at the price of $\frac{c_L}{1-\eta}$ when $\frac{c_L}{1-\eta} > (1-r_L)p + r_L s$, in which case $\Pi_1(0,0) = 0$. If we have $(\delta_1, \delta_2) = (1,0)$ in equilibrium, then in equilibrium, we must have $r_1 = r_H$ and manufacturer 1 will receive d from each buyer at the price of (approximately) $\frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H}$, in which case $\Pi_1(1,0) = (1-\eta)r_H(\frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} - \frac{c_H}{1-\eta})D > 0$. Therefore, when $\frac{c_L}{1-\eta} > (1-r_L)p + r_L s$, we have $\Pi_1(1,0) > \Pi_1(0,0)$. When $\frac{c_L}{1-\eta} \leq (1-r_L)p + r_L s$,

$$\begin{aligned}\Pi_1(0,0) - \Pi_1(1,0) =& (1-\eta)r_L\left((1-r_L)p + r_L s - \frac{c_L}{1-\eta}\right)D - (1-\eta)r_H\left(\frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} - \frac{c_H}{1-\eta}\right)D \\ =& (1-\eta)\left(\frac{r_H c_H - r_L c_L}{1-\eta} - (r_H - r_L)p - r_L\left(\frac{c_L}{1-\eta} - ((1-r_L)p + r_L s)\right)\right)D,\end{aligned}$$

and thus we have $\Pi_1(0,0) < \Pi_1(1,0)$ if $\frac{r_H c_H - r_L c_L}{1-\eta} < (r_H - r_L)p + r_L\big(\frac{c_L}{1-\eta} - ((1-r_L)p + r_L s)\big)$. To summarize, since $\frac{c_L}{1-\eta} > (1-r_L)p + r_L s \iff (r_H - r_L)p + r_L\left(\frac{c_L}{1-\eta} - ((1-r_L)p + r_L s)\right) > (r_H - r_L)p$, we have $\Pi_1(0,0) < \Pi_1(1,0)$ when (1) $\frac{c_H}{1-\eta} > (1-r_L)p + r_L s$ or $\frac{c_L}{1-\eta} > (1-r_H)p + r_H s$, and (2) $\frac{r_H c_H - r_L c_L}{1-\eta} < \min\{(r_H - r_L)p + r_L\big(\frac{c_L}{1-\eta} - ((1-r_L)p + r_L s)\big), (r_H - r_L)p\}$, where condition (2) is equivalent to $c_H < \bar{\bar{c}}_H$. Otherwise, under condition (c), we have $\Pi_1(0,0) \geq \Pi_1(1,0)$.

(ii) We next compare $\Pi_1(0,1)$ with $\Pi_1(1,1)$. Consider the following two cases:

(a) Suppose $\frac{c_H}{1-\eta} \leq (1-r_H)p + r_H s$ (which implies $\frac{c_H}{1-\eta} < (1-r_L)p + r_L s$ and $\frac{c_L}{1-\eta} < (1-r_H)p + r_H s$). From our analysis in Step 2, we know that if we have $(\delta_1, \delta_2) = (0,1)$ in equilibrium, then in equilibrium, we must have $r_1 = r_L$ and manufacturer 1 will receive d from each buyer at the price of $(1-r_H)p + r_H s$. Therefore, we have $\Pi_1(0,1) = (1-\eta)r_L((1-r_H)p + r_H s - \frac{c_L}{1-\eta})D$. If we have $(\delta_1, \delta_2) = (1,1)$ in equilibrium, then in equilibrium, we must have $r_1 = r_H$ and manufacturer 1 will receive d from each buyer at the price of $(1-r_H)p+r_H s$. Therefore, we have $\Pi_1(1,1) = (1-\eta)r_H((1-r_H)p + r_H s - \frac{c_H}{1-\eta})D$. Then, we have $\Pi_1(0,1) > \Pi_1(1,1)$ because

$$\begin{aligned}\Pi_1(0,1) - \Pi_1(1,1) &= (1-\eta)r_L\left((1-r_H)p + r_H s - \frac{c_L}{1-\eta}\right)D - (1-\eta)r_H\left((1-r_H)p + r_H s - \frac{c_H}{1-\eta}\right)D \\ &= (1-\eta)\left(\frac{r_H c_H - r_L c_L}{1-\eta} - (r_H - r_L)\left((1-r_H)p + r_H s\right)\right)D > 0,\end{aligned}$$

where the inequality follows from $\frac{r_H c_H - r_L c_L}{(1-\eta)(r_H - r_L)} > (1-r_H)p + r_H s$. Therefore, when $\frac{c_H}{1-\eta} \leq (1-r_H)p+r_H s$, there must be no equilibrium that satisfies $(\delta_1, \delta_2) = (1,1)$, while $(\delta_1, \delta_2) = (0,1)$ remains a possible certification status profile that can be sustained in equilibrium.

(b) Suppose $\frac{c_H}{1-\eta} > (1-r_H)p + r_H s$. From our analysis in Step 2, we know that if we have $(\delta_1, \delta_2) = (1,1)$ in equilibrium, then in equilibrium, we must have $r_1 = r_H$ and $w_1 = \frac{c_H}{1-\eta}$. In this case, manufacturer 1 has zero profit. By the tie-breaking rule, manufacturer 1 will deviate by choosing a lower reliability level. Therefore, there must be no equilibrium that satisfies $(\delta_1, \delta_2) = (1,1)$, while $(\delta_1, \delta_2) = (0,1)$ remains a possible certification status profile that can be sustained in equilibrium.

Combining all cases, we conclude that $(\delta_1, \delta_2) = (1,0)$ and $(\delta_1, \delta_2) = (0,1)$ remain the only possible certification status profiles that can be sustained in equilibrium when (1) $\frac{c_H}{1-\eta} > (1-r_L)p + r_L s$ or $\frac{c_L}{1-\eta} > (1-r_H)p + r_H s$, and (2) $c_H < \bar{\bar{c}}_H$; and that $(\delta_1, \delta_2) = (0,0)$ remains the only possible certification status profile that can be sustained in equilibrium otherwise. Moreover, following our analysis in Steps 2 and 3, it can be verified that for each of these certification status profiles, the corresponding strategy profile characterized in Step 2 together with beliefs consistent with the strategy profile can be sustained as an equilibrium (i.e., no one has an incentive to deviate and beliefs are consistent with the strategy profile).

We next show that condition (1) above is redundant by proving that $c_H < \bar{\bar{c}}_H \implies \frac{c_L}{1-\eta} > (1-r_H)p+r_H s$. As shown at the beginning of this step, under Assumption 1, we always have $\frac{r_H c_H - r_L c_L}{(1-\eta)(r_H - r_L)} - ((1-r_L)p+r_L s) > 0$, i.e., $c_H > \frac{r_L}{r_H}c_L + (1-\frac{r_L}{r_H})\bar{\bar{c}}_L(r_L)$. Therefore, $c_H < \bar{\bar{c}}_H \implies \frac{r_L}{r_H}c_L + (1-\frac{r_L}{r_H})\bar{\bar{c}}_L(r_L) < \bar{\bar{c}}_H$, i.e.,

$$\frac{r_L c_L + (1-\eta)(r_H - r_L)\left((1-r_L)p + r_L s\right)}{r_H} < \min\{\frac{2r_L c_L + (1-\eta)\left((r_L^2 - 2r_L + r_H)p - r_L^2 s\right)}{r_H}, \frac{r_L c_L + (1-\eta)(r_H - r_L)p}{r_H}\}.$$

Since $(1-r_L)p + r_L s < p$, we have $\frac{r_L c_L + (1-\eta)(r_H - r_L)((1-r_L)p + r_L s)}{r_H} < \frac{r_L c_L + (1-\eta)(r_H - r_L)p}{r_H}$. Therefore,

$$\begin{aligned}&\frac{r_L}{r_H}c_L + \left(1 - \frac{r_L}{r_H}\right)\bar{\bar{c}}_L(r_L) < \bar{\bar{c}}_H \\ \iff &\frac{r_L c_L + (1-\eta)(r_H - r_L)\left((1-r_L)p + r_L s\right)}{r_H} < \frac{2r_L c_L + (1-\eta)\left((r_L^2 - 2r_L + r_H)p - r_L^2 s\right)}{r_H} \\ \iff &\frac{c_L}{1-\eta} > (1-r_H)p + r_H s.\end{aligned}$$

To summarize, when $c_H < \bar{\bar{c}}_H$ (i.e., $a < \bar{\bar{a}}$), one manufacturer chooses r_H while the other chooses r_L in equilibrium. Otherwise, both choose r_L in equilibrium. The corresponding manufacturer pricing decisions are characterized in Step 2, and buyer sourcing decisions are characterized in Lemma 1. □

Proof of Proposition 1. To prove the proposition, we first obtain the expected shortage cost for patients with and without certification, respectively. Recall that we focus on $a < \bar{\bar{a}}$. Then, with certification, based on Lemma 3, we know that one manufacturer chooses r_H while the other manufacturer chooses r_L; buyers source a total of D from the high-reliability manufacturer and nothing from the low-reliability manufacturer. Then, there is a probability $1 - r_H$ of experiencing a shortage quantity of D, leading to an expected shortage cost of $\gamma(1 - r_H)D^2$. Without certification, based on Lemma 2, we have:

(i) When $c_L \le \bar{\bar{c}}_L(r_L)$, both manufacturers choose r_L, and buyers source D from each manufacturer. Then, there is a probability $(1 - r_L)^2$ of experiencing a shortage quantity of D (i.e., when both manufacturers are under disruption), leading to an expected shortage cost of $\gamma(1 - r_L)^2 D^2$.

(ii) When $c_L > \bar{\bar{c}}_L(r_L)$, both manufacturers choose r_L, and buyers source $\frac{D}{2}$ from each manufacturer. Then, there is a probability $2r_L(1 - r_L)$ of experiencing a shortage quantity of $\frac{D}{2}$ (i.e., when exactly one manufacturers is under disruption) and a probability $(1 - r_L)^2$ of experiencing a shortage quantity of D (i.e., when both manufacturers are under disruption). Therefore, in this case, the expected shortage cost is given by $2\gamma r_L(1 - r_L)(\frac{D}{2})^2 + \gamma(1 - r_L)^2 D^2 = \gamma \frac{r_L^2 - 3r_L + 2}{2} D^2$.

Therefore, when $c_L \le \bar{\bar{c}}_L(r_L)$, certification leads to a strictly higher expected shortage cost when $r_H < 2r_L - r_L^2$ because $1 - r_H > (1 - r_L)^2 \iff r_H < 2r_L - r_L^2$.

When $c_L > \bar{\bar{c}}_L(r_L)$, certification leads to a strictly higher expected shortage cost when $r_H < 2r_L - r_L^2 - \frac{1}{2} r_L(1 - r_L)$ because $1 - r_H > \frac{r_L^2 - 3r_L + 2}{2} \iff r_H < \frac{3}{2} r_L - \frac{1}{2} r_L^2 = 2r_L - r_L^2 - \frac{1}{2} r_L(1 - r_L)$.

To conclude, certification leads to a strictly higher expected shortage cost compared to no certification if and only if $r_H < \bar{r}_H = 2r_L - r_L^2 - \frac{r_L(1-r_L)}{2} \mathbf{1}[c_L > \bar{\bar{c}}_L(r_L)]$, which completes the proof. □

Proof of Proposition 2. To prove the proposition, we first obtain the expected manufacturer profit with and without certification, respectively. Recall that we focus on $a < \bar{\bar{a}}$, which is equivalent to $c_H < \bar{\bar{c}}_H$, where $c_H = c_L + a(r_H - r_L)$. Then, with certification, based on Lemma 3, one manufacturer chooses r_H, and the high-reliability manufacturer receives a demand D from the buyers at the price of (approximately) $\frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H}$ and the low-reliability manufacturer receives nothing. Then, the expected average manufacturer profit is $\frac{1}{2} r_H \big((1-\eta) \frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} - c_H\big) D - f = \big(r_L c_L - r_H c_H + (1-\eta)(r_H - r_L)p\big) \frac{D}{2} - f$. Without certification, based on Lemma 2, we have:

(i) When $c_L \le \bar{\bar{c}}_L(r_L)$, each manufacturer chooses r_L and receives D at the price of $(1 - r_L)p + r_L s$, leading to an expected manufacturer profit of $r_L\left((1-\eta)\left((1 - r_L)p + r_L s\right) - c_L\right) D$.

(ii) When $c_L > \bar{\bar{c}}_L(r_L)$, each manufacturer chooses r_L and receives $\frac{D}{2}$ at the price of $\frac{c_L}{1-\eta}$, leading to a zero profit.

Therefore, when $c_L \le \bar{\bar{c}}_L(r_L)$, certification leads to a strictly lower expected manufacturer profit than if

$$
\begin{aligned}
&(r_L c_L - r_H c_H + (1-\eta)(r_H - r_L)p)\,\frac{D}{2} - f - r_L\left((1-\eta)\left((1-r_L)p + r_L s\right) - c_L\right) D < 0 \\
\iff\ & c_H > \frac{r_L c_L + (1-\eta)(r_H - r_L)p - \frac{2f}{D} - 2r_L\left((1-\eta)\left((1-r_L)p + r_L s\right) - c_L\right)}{r_H} \\
\iff\ & c_H > \frac{r_L c_L + (1-\eta)(r_H - r_L)p - \frac{2f}{D} - 2r_L(\bar{\bar{c}}_L(r_L) - c_L)}{r_H}.
\end{aligned}
$$

When $c_L > \bar{\bar{c}}_L(r_L)$, certification leads to a strictly lower expected manufacturer's profit if $(r_L c_L - r_H c_H + (1-\eta)(r_H - r_L)p)\frac{D}{2} - f < 0 \iff c_H > \frac{r_L c_L + (1-\eta)(r_H - r_L)p - \frac{2f}{D}}{r_H}$.

To conclude, certification leads to a strictly lower expected manufacturer profit if and only if $c_H > \frac{r_L c_L + (1-\eta)(r_H - r_L)p - \frac{2f}{D} - 2r_L(\bar{\bar{c}}_L(r_L) - c_L)^+}{r_H} \iff a > \bar{a}$, which completes the proof. □

Proof of Proposition 3. To prove the proposition, we first derive each buyer's expected cost with and without certification. Recall that we focus on $a < \bar{\bar{a}}$. Then, with certification, based on Lemma 3, one manufacturer chooses r_H, and each buyer source d from the high-reliability manufacturer at the price of (approximately) $\frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H}$ and nothing from the low-reliability manufacturer. There is a probability r_H that each buyer pays the high-reliability manufacturer for the delivered quantity d, and a probability $1 - r_H$ of paying a shortage penalty pd. This leads to an expected buyer cost of $r_H \frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} d + (1 - r_H)pd = (1 - r_L)pd + r_L \frac{c_L}{1-\eta} d$. Without certification, based on Lemma 2, we have:

(i) When $c_L \le \bar{\bar{c}}_L(r_L)$, both manufacturer chooses r_L, and each buyer sources d from each manufacturer at the price of $(1 - r_L)p + r_L s$. This leads to an expected buyer cost of $r_L^2\left(2\left((1-r_L)p + r_L s\right) - s\right) d + 2r_L(1 - r_L)\left((1-r_L)p + r_L s\right) d + (1 - r_L)^2 pd = (1 - r_L^2)pd + r_L^2 sd$.

(ii) When $c_L > \bar{\bar{c}}_L(r_L)$, both manufacturer chooses r_L, and each buyer source $\frac{d}{2}$ from each manufacturer at the price of $\frac{c_L}{1-\eta}$. This leads to an expected buyer cost of $r_L^2 \frac{c_L}{1-\eta} d + 2r_L(1-r_L)(\frac{c_L}{1-\eta} + p)\frac{d}{2} + (1-r_L)^2 pd = (1 - r_L)pd + r_L \frac{c_L}{1-\eta} d$.

Therefore, when $c_L \le \bar{\bar{c}}_L(r_L)$, certification leads to a lower expected buyer cost because

$$
(1-r_L)pd + r_L \frac{c_L}{1-\eta} d - \left((1 - r_L^2)p + r_L^2 s\right) d = r_L\left(\frac{c_L}{1-\eta} - \left((1-r_L)p + r_L s\right)\right) d = \frac{r_L}{1-\eta}\left(c_L - \bar{\bar{c}}_L(r_L)\right) d \le 0.
$$

When $c_L > \bar{\bar{c}}_L(r_L)$, certification leads to the same expected buyer as without certification. To conclude, certification leads to a lower expected buyer cost, which completes the proof. □

Proof of Proposition 4. To prove the proposition, we first obtain the expected GPO profit with and without certification, respectively. Recall that we focus on $a < \bar{\bar{a}}$. Then, with certification, based on Lemma 3, one manufacturer chooses r_H, and the high-reliability manufacturer has a probability of r_H delivering D demands to the buyers at the price of (approximately) $\frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H}$, leading to an expected revenue of $r_H \frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} D$ for the manufacturer. On the other hand, the low-reliability manufacturer receives nothing from the buyers. Therefore, the expected GPO profit is given by $\eta r_H \frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} D = \eta r_L \frac{c_L}{1-\eta} D + \eta(r_H - r_L)pD$. Without certification, based on Lemma 2, we have:

(i) When $c_L \le \bar{\bar{c}}_L(r_L)$, each manufacturer chooses r_L, and with probability r_L, delivers D to the buyers at the price $(1-r_L)p + r_L s$, leading to an expected GPO profit of $2\eta r_L\left((1-r_L)p + r_L s\right)D$.

(ii) When $c_L > \bar{\bar{c}}_L(r_L)$, each manufacturer chooses r_L, and with probability r_L, delivers $\frac{D}{2}$ to the buyers at the price $\frac{c_L}{1-\eta}$, leading to an expected GPO profit of $2\eta r_L \frac{c_L}{1-\eta}\frac{D}{2} = \eta r_L \frac{c_L}{1-\eta} D$.

Therefore, when $c_L \le \bar{\bar{c}}_L(r_L)$, certification leads to a strictly lower expected GPO profit if $\eta r_L \frac{c_L}{1-\eta} D + \eta(r_H - r_L)pD < 2\eta r_L((1-r_L)p + r_L s)D \iff c_L < 2\bar{\bar{c}}_L(r_L) - (1-\eta)(\frac{r_H}{r_L} - 1)p$. When $c_L > \bar{\bar{c}}_L(r_L)$, since $\eta r_L \frac{c_L}{1-\eta} D + \eta(r_H - r_L)pD > \eta r_L \frac{c_L}{1-\eta} D$, certification leads to a higher expected GPO profit.

To conclude, certification leads to a strictly lower GPO profit if and only if $c_L < \bar{c}_L = (2\bar{\bar{c}}_L(r_L) - (1-\eta)(\frac{r_H}{r_L} - 1)p)\mathbf{1}[c_L \le \bar{\bar{c}}_L(r_L)]$, which completes the proof. □

Proof of Corollary 1. As shown in Propositions 1-4, certification leads to a lower expected patients' shortage cost when $r_H \ge \bar{r}_H$ (Proposition 1), a higher expected manufacturer profit when $a \le \bar{a}$ (Proposition 2), a lower expected buyer cost (Proposition 3), and a higher expected GPO profit when $c_L \ge \bar{c}_L$ (Proposition 4).

Therefore, when all of the above conditions hold, that is, when $r_H \ge \bar{r}_H, a \le \bar{a}$ and $c_L \ge \bar{c}_L$, certification achieves Pareto improvement for patients, manufacturers, buyers, and the GPO. □

A.3 Proofs of Results in Section 5

Proof of Proposition 5. We prove this proposition by first characterizing the equilibrium decisions under certification and subsidies (see Table A.1). When a certified high-reliability manufacturer is offered a subsidy k, the subsidy effectively reduces the production cost for the high-reliability case from $c_i(r_i)$ to $c_i(r_i) - k$, where $r_i \ge r_H$. Accordingly, the expected profit for a manufacturer, say, manufacturer 1, is:

$$\Pi_1^{\text{Certified}}(r_1, w_1|\delta_2, w_2) = \begin{cases} \sum_{j=1}^{N} r_1\left((1-\eta)w_1 - c_1(r_1)\right) Q_{1j}((\delta_1,\delta_2),(w_1,w_2)) - f & \text{if } r_1 < r_H, \\ \sum_{j=1}^{N} r_1\left((1-\eta)w_1 - c_1(r_1) + k\right) Q_{1j}((\delta_1,\delta_2),(w_1,w_2)) - f & \text{if } r_1 \ge r_H. \end{cases}$$

With the above change, Steps 1 and 2 of the proof of Lemma 3 remain valid. That is, buyers' sourcing strategies remain unchanged, and for each certification status profile (δ_1, δ_2) that may arise in equilibrium, the necessary conditions that the corresponding equilibrium reliability levels and prices must satisfy also continue to hold (except that if $\delta_i = 1$, then $c_i(r_i)$ should be replaced with $c_i(r_i) - k$).

Therefore, to characterize the equilibrium, it remains to check how Step 3 of the proof of Lemma 3 needs to be revised due to subsidies. As in the proof of Lemma 3, let $\Pi_i(\delta_1, \delta_2)$ denote the manufacturer i's expected profit given the certification status profile (δ_1, δ_2), where the corresponding reliability and pricing choices satisfy the equilibrium conditions characterized in Step 2 of the proof of Lemma 3.

(i) We first compare $\Pi_1(0,0)$ with $\Pi_1(1,0)$. Consider the following three cases:

(a) Suppose $\frac{c_H-k}{1-\eta} \le (1-r_L)p + r_L s$, $\frac{c_L}{1-\eta} \le (1-r_H)p + r_H s$. Then, following a similar analysis as in case (i.a) of Step 3 in the proof of Lemma 3, we have

$$\Pi_1(0,0) - \Pi_1(1,0) = (1-\eta)\left(\frac{r_H(c_H-k) - r_L c_L}{1-\eta} - (r_H - r_L)\left((1-r_L)p + r_L s\right)\right) D.$$

Therefore, we have $\Pi_1(0,0) < \Pi_1(1,0)$ if $\frac{r_H(c_H-k)-r_L c_L}{1-\eta} < (r_H - r_L)((1-r_L)p + r_L s)$, which is equivalent to $\frac{c_H-k}{1-\eta} < \frac{r_L}{r_H}\frac{c_L}{1-\eta} + (1 - \frac{r_L}{r_H})\left((1-r_L)p + r_L s\right)$, and we have $\Pi_1(0,0) \ge \Pi_1(1,0)$ otherwise.

Given that $\frac{c_L}{1-\eta} \le (1-r_H)p + r_H s < (1-r_L)p + r_L s$, we know that $\frac{r_L}{r_H}\frac{c_L}{1-\eta} + (1-\frac{r_L}{r_H})((1-r_L)p + r_L s) < (1-r_L)p + r_L s$. Further, recall that $\bar{\bar{c}}_L(r_L) = (1-\eta)\big((1-r_L)p + r_L s\big)$. Then, we have $\Pi_1(0,0) < \Pi_1(1,0)$ when $\frac{c_L}{1-\eta} \le (1-r_H)p + r_H s$ and $c_H - k < \frac{r_L}{r_H}c_L + (1-\frac{r_L}{r_H})\bar{\bar{c}}_L(r_L)$ (note that the latter condition holds only when k is sufficiently high because as shown in the proof of Lemma 3, we have $\frac{r_H c_H - r_L c_L}{(1-\eta)(r_H - r_L)} - ((1-r_L)p + r_L s) > 0$, i.e., $c_H > \frac{r_L}{r_H}c_L + (1-\frac{r_L}{r_H})\bar{\bar{c}}_L(r_L)$). Otherwise, under condition (a), we have $\Pi_1(0,0) \ge \Pi_1(1,0)$.

(b) Suppose (1) $\frac{c_H-k}{1-\eta} > (1-r_L)p + r_L s$ or $\frac{c_L}{1-\eta} > (1-r_H)p + r_H s$, and (2) $\frac{r_H(c_H-k)-r_L c_L}{1-\eta} \ge (r_H - r_L)p$. Then, following a similar analysis as in case (i.b) of Step 3 in the proof of Lemma 3, there must be no equilibrium that satisfies $(\delta_1, \delta_2) = (1,0)$, while $(\delta_1, \delta_2) = (0,0)$ remains a possible certification status profile that can be sustained in equilibrium.

(c) Suppose (1) $\frac{c_H-k}{1-\eta} > (1-r_L)p + r_L s$ or $\frac{c_L}{1-\eta} > (1-r_H)p + r_H s$, and (2) $\frac{r_H(c_H-k)-r_L c_L}{1-\eta} < (r_H - r_L)p$. Then, following a similar analysis as in case (i.c) of Step 3 in the proof of Lemma 3, we have $\Pi_1(0,0) < \Pi_1(1,0)$ when (1) $\frac{c_H-k}{1-\eta} > (1-r_L)p + r_L s$ or $\frac{c_L}{1-\eta} > (1-r_H)p + r_H s$, and (2) $c_H - k < \bar{\bar{c}}_H$, where $\bar{\bar{c}}_H$ is defined in the proof of Lemma 3. Otherwise, under condition (c), we have $\Pi_1(0,0) \ge \Pi_1(1,0)$.

Combining all cases, we conclude that $\Pi_1(0,0) < \Pi_1(1,0)$ if and only if (1) $\frac{c_L}{1-\eta} \le (1-r_H)p + r_H s$, $c_H - k < \frac{r_L}{r_H}c_L + (1-\frac{r_L}{r_H})\bar{\bar{c}}_L(r_L)$, or (2) $\frac{c_L}{1-\eta} > (1-r_H)p + r_H s$, $c_H - k < \bar{\bar{c}}_H$, or (3) $\frac{c_H-k}{1-\eta} > (1-r_L)p + r_L s$, $c_H - k < \bar{\bar{c}}_H$. Moreover, we next show that condition (3) is redundant by proving that whenever $c_H - k < \bar{\bar{c}}_H$, either (1) or (2) must hold. If $\frac{c_L}{1-\eta} \le (1-r_H)p + r_H s$, then, by simple algebra, we have $\frac{r_L}{r_H}c_L + (1-\frac{r_L}{r_H})\bar{\bar{c}}_L(r_L) \ge \bar{\bar{c}}_H$. Therefore, we have $c_H - k < \bar{\bar{c}}_H \le \frac{r_L}{r_H}c_L + (1-\frac{r_L}{r_H})\bar{\bar{c}}_L(r_L)$, and thus condition (1) holds. If $\frac{c_L}{1-\eta} > (1-r_H)p + r_H s$, then given $c_H - k < \bar{\bar{c}}_H$, condition (2) holds.

Further, since $\frac{c_L}{1-\eta} \le (1-r_H)p + r_H s \iff \frac{r_L}{r_H}c_L + (1-\frac{r_L}{r_H})\bar{\bar{c}}_L(r_L) \ge \bar{\bar{c}}_H$, we conclude that $\Pi_1(0,0) < \Pi_1(1,0)$ if and only if $c_H - k < \max\{\frac{r_L}{r_H}c_L + (1-\frac{r_L}{r_H})\bar{\bar{c}}_L(r_L), \bar{\bar{c}}_H\}$. (Note that when $k=0$, this condition is equivalent to $c_H < \bar{\bar{c}}_H$ because as discussed in the proof of Lemma 3, under Assumption 1, we always have $c_H > \frac{r_L}{r_H}c_L + (1-\frac{r_L}{r_H})\bar{\bar{c}}_L(r_L)$.)

(ii) We next compare $\Pi_1(0,1)$ with $\Pi_1(1,1)$. Consider the following three cases:

(a) Suppose $\frac{c_H-k}{1-\eta} < (1-r_H)p + r_H s$ (which implies $\frac{c_H-k}{1-\eta} < (1-r_L)p + r_L s$) and $\frac{c_L}{1-\eta} < (1-r_H)p + r_H s$. Then, following a similar analysis as in case (ii.a) of Step 3 in the proof of Lemma 3, we have

$$\Pi_1(0,1) - \Pi_1(1,1) = (1-\eta)\left(\frac{r_H(c_H-k) - r_L c_L}{1-\eta} - (r_H - r_L)\left((1-r_H)p + r_H s\right)\right) D.$$

Therefore, we have $\Pi_1(0,1) < \Pi_1(1,1)$ if $\frac{r_H(c_H-k)-r_L c_L}{1-\eta} < (r_H - r_L)\big((1-r_H)p + r_H s\big)$, which is equivalent to $\frac{c_H-k}{1-\eta} < \frac{r_L}{r_H}\frac{c_L}{1-\eta} + (1-\frac{r_L}{r_H})\big((1-r_H)p + r_H s\big)$, and we have $\Pi_1(0,1) \geq \Pi_1(0,0)$ otherwise.

(b) Suppose $\frac{c_H-k}{1-\eta} < (1-r_H)p + r_H s$ (which implies $\frac{c_H-k}{1-\eta} < (1-r_L)p + r_L s$) and $\frac{c_L}{1-\eta} \geq (1-r_H)p + r_H s$. In this case, we have $\frac{r_H(c_H-k)-r_L c_L}{1-\eta} < (r_H - r_L)p$. If we have $(\delta_1, \delta_2) = (0,1)$ in equilibrium, then following a similar analysis as in Step 2 of the proof of Lemma 3, we know that in equilibrium, we must have $r_1 = r_L$ and manufacturer 1 will receive nothing from each buyer. Therefore, we have $\Pi_1(0,1) = 0$. If we have $(\delta_1, \delta_2) = (1,1)$ in equilibrium, then in equilibrium, we must have $r_1 = r_H$ and manufacturer 1 will receive d from each buyer at the price of $(1-r_H)p + r_H s$. Therefore, we have $\Pi_1(1,1) = (1-\eta) r_H((1-r_H)p + r_H s - \frac{c_H-k}{1-\eta})D$. Therefore, we have $\Pi_1(0,1) < \Pi_1(1,1)$.

(c) Suppose $\frac{c_H-k}{1-\eta} \geq (1-r_H)p + r_H s$. Then, following a similar analysis as in Step 2 of the proof of Lemma 3, we know that there must be no equilibrium that satisfies $(\delta_1, \delta_2) = (1,1)$, while $(\delta_1, \delta_2) = (0,1)$ remains a possible certification status profile that can be sustained in equilibrium.

Recall that $\bar{\bar{c}}_L(r_H) = (1-\eta)\left((1-r_H)p + r_H s\right)$. Since $\frac{c_L}{1-\eta} \leq (1-r_H)p + r_H s \iff \frac{r_L}{r_H} c_L + (1 - \frac{r_L}{r_H})\bar{\bar{c}}_L(r_H) \leq \bar{\bar{c}}_L(r_H)$, combining all cases, we conclude that $\Pi_1(0,1) < \Pi_1(1,1)$ if and only if $c_H - k < \min\{\frac{r_L}{r_H} c_L + (1-\frac{r_L}{r_H})\bar{\bar{c}}_L(r_H), \bar{\bar{c}}_L(r_H)\}$.

Let $A = \min\{\frac{r_L}{r_H} c_L + (1-\frac{r_L}{r_H})\bar{\bar{c}}_L(r_H), \bar{\bar{c}}_L(r_H)\}$ and $B = \max\{\frac{r_L}{r_H} c_L + (1-\frac{r_L}{r_H})\bar{\bar{c}}_L(r_L), \bar{\bar{c}}_H\}$. Then, following a similar reasoning as in the proof of Lemma 3, we conclude that in equilibrium, both manufacturers choose r_H when $c_H - k < A$, one manufacturer chooses r_H while the other chooses r_L when $A \leq c_H - k < B$, and both choose r_L otherwise. A summary of the equilibrium decisions is presented in Table A.1.

Condition	r_i^*, r_{-i}^*	w_i^*, w_{-i}^*	$Q_{ij}^*, Q_{(-i)j}^*$
$c_H - k < A$	r_H, r_H	$(1-r_H)p + r_H s, (1-r_H)p + r_H s$	d, d
$A \leq c_H - k < B$, $c_L \leq \bar{\bar{c}}_L(r_H)$	r_H, r_L	$(1-r_L)p + r_L s, (1-r_H)p + r_H s$	d, d
$A \leq c_H - k < B$, $c_L > \bar{\bar{c}}_L(r_H)$	r_H, r_L	$\frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} - \epsilon, \frac{c_L}{1-\eta}$	$d, 0$
$c_H - k \geq B$, $c_L \leq \bar{\bar{c}}_L(r_L)$	r_L, r_L	$(1-r_L)p + r_L s, (1-r_L)p + r_L s$	d, d
$c_H - k \geq B$, $c_L > \bar{\bar{c}}_L(r_L)$	r_L, r_L	$\frac{c_L}{1-\eta}, \frac{c_L}{1-\eta}$	$\frac{1}{2}d, \frac{1}{2}d$

Notes: $A = \min\{\frac{r_L}{r_H} c_L + (1-\frac{r_L}{r_H})\bar{\bar{c}}_L(r_H), \bar{\bar{c}}_L(r_H)\}$, $B = \max\{\frac{r_L}{r_H} c_L + (1-\frac{r_L}{r_H})\bar{\bar{c}}_L(r_L), \bar{\bar{c}}_H\}$.

Table A.1. Equilibrium decisions under certification and subsidies.

By comparing the equilibrium decisions under certification with and without subsidies (see Table A.1 and Table 3, respectively), we can see that subsidies lead to a strictly lower expected shortage cost for patients when $k > c_H - A$, because under this condition, subsidies motivate both manufacturers to choose high reliability and each buyer to source d from each manufacturer (in this case, there is only a probability $(1-r_H)^2$ of experiencing a shortage of D). We consider $c_H - k \geq A$ in the remainder of our analysis.

(i) Suppose $c_H \geq \bar{\bar{c}}_H$ and $c_L \leq \bar{\bar{c}}_L(r_L)$. Then, without subsidies, as shown in Lemma 3, both manufacturers choose $\delta_1 = 0$ and each buyer sources d from each manufacturer. With subsidies, if $c_H - k \geq B$, then the equilibrium decisions are the same as the no subsidy case (see Table A.1; as discussed earlier, $c_H \geq B$ is equivalent to $c_H \geq \bar{\bar{c}}_H$); if $A \leq c_H - k < B$, the equilibrium decisions as follows:

  (a) One manufacturer chooses r_H while the other chooses r_L and each buyer source d from each manufacturer if $c_L \leq \bar{\bar{c}}_L(r_H)$.

  (b) One manufacturer chooses r_H while the other chooses r_L, and each buyer sources d from the high-reliability manufacturer and nothing from the low-reliability manufacturer if $c_L > \bar{\bar{c}}_L(r_H)$.

  Then, it is straightforward that subsidies lead to a lower expected shortage cost when $c_L \leq \bar{\bar{c}}_L(r_H)$. When $c_L > \bar{\bar{c}}_L(r_H)$, following a similar analysis as in the proof of Proposition 1, we have that subsidies lead to a strictly higher expected shortage cost than the no subsidy case if $r_H < \bar{r}_H$.

(ii) Suppose $c_H \geq \bar{\bar{c}}_H$ and $c_L > \bar{\bar{c}}_L(r_L)$ (which implies $c_L > \bar{\bar{c}}_L(r_H)$). Then, without subsidies, as shown in Lemma 3, both manufacturers choose $\delta_1 = 0$ and each buyer sources $\frac{d}{2}$ from each manufacturer. With subsidies, if $c_H - k \geq B$, then the equilibrium decisions are the same as the no subsidy case; if $A \leq c_H - k < B$, then one manufacturer chooses r_H while the other chooses r_L, and each buyer sources d from the high-reliability manufacturer and nothing from the low-reliability manufacturer. Then, following a similar analysis as in the proof of Proposition 1, we have that subsidies lead to a strictly higher expected shortage cost than the no subsidy case if $r_H < \bar{r}_H$.

(iii) Suppose $c_H < \bar{\bar{c}}_H$ (which implies $c_H - k < B$ for any $k \geq 0$). Then, without subsidies, as shown in Lemma 3, one manufacturer chooses r_H while the other chooses r_L, and each buyer sources d from the high-reliability manufacturer and nothing from the low-reliability manufacturer. With subsidies, if $A \leq c_H - k < B$, the equilibrium decisions as follows:

  (a) One manufacturer chooses r_H while the other chooses r_L and each buyer source d from each manufacturer if $c_L \leq \bar{\bar{c}}_L(r_H)$.

  (b) One manufacturer chooses r_H while the other chooses r_L, and each buyer sources d from the high-reliability manufacturer and nothing from the low-reliability manufacturer if $c_L > \bar{\bar{c}}_L(r_H)$.

  Then, it is straightforward that subsidies lead to a (weakly) lower expected shortage cost.

Combining all cases, we conclude that subsidies lead to a strictly higher expected shortage cost if and only if $r_H < \bar{r}_H$, $c_H \geq \bar{\bar{c}}_H$ (i.e., which is equivalent to $a \geq \bar{\bar{a}}$), $c_L > \bar{\bar{c}}_L(r_H)$, and $c_H - B < k \leq c_H - A$. Since $c_L > \bar{\bar{c}}_L(r_H)$ implies $\frac{r_L}{r_H}c_L + (1 - \frac{r_L}{r_H})\bar{\bar{c}}_L(r_H) > \bar{\bar{c}}_L(r_H)$ and $\frac{r_L}{r_H}c_L + (1 - \frac{r_L}{r_H})\bar{\bar{c}}_L(r_L) < \bar{\bar{c}}_H$ (as shown in the proof of Lemma 3, we have $c_L > \bar{\bar{c}}_L(r_H) \iff \frac{r_L}{r_H}c_L + (1 - \frac{r_L}{r_H})\bar{\bar{c}}_L(r_L) < \bar{\bar{c}}_H$), we have $A = \bar{\bar{c}}_L(r_H)$ and $B = \bar{\bar{c}}_H = c_L + \bar{\bar{a}}(r_H - r_L)$, which completes the proof. □

Supplemental Appendix for "Mitigating Shortages of Generic Drugs: The Role of Reliability Certification"

A Endogenous Certification Decisions

In our main model, we consider a setting where under certification, whether the reliability level of each manufacturer has reached the certification threshold r_H is public information. In this section, we extend our model to consider a setting where manufacturers endogenously determine whether to participate in certification, and demonstrate that our key insights remain qualitatively similar. Specifically, we consider that in the first step of the game, each manufacturer determines both its reliability level and whether to participate in certification. If a manufacturer participates, it is publicly revealed whether its chosen reliability level has reached r_H. Otherwise, its reliability is private information. After the manufacturers' decisions, the rest of the sequence of events is identical to that in the main model (as shown in Figure 1). Only manufacturers that participate in certification pay the fixed cost of certification f.

Recall that $\bar{\bar{a}}$ denotes the threshold on a above which no manufacturer improves reliability when both manufacturers participate in certification. Let a' denote a threshold that satisfy $a' \leq \bar{\bar{a}}$. Then, we have:

Lemma A.1. *When manufacturers' certification decisions are endogenous, the equilibrium decisions of all players are characterized as follows:*
(i) If $a \geq a'$, neither manufacturer participates in certification and neither improves reliability. Further, if $c_L \leq \bar{\bar{c}}_L(r_L)$, then both manufacturers set a wholesale price of $(1-r_L)p+r_Ls$, and each buyer sources d from each manufacturer; otherwise (i.e., $c_L > \bar{\bar{c}}_L(r_L)$), both manufacturers set a wholesale price of $\frac{c_L}{1-\eta}$, and each buyer sources $\frac{1}{2}d$ from each manufacturer.
(ii) If $a < a'$, one manufacturer participates in certification, chooses a reliability level of r_H, and sets a wholesale price of $\frac{r_L\frac{c_L}{1-\eta}+(r_H-r_L)p}{r_H} - \epsilon$. The other (non-certified) manufacturer retains at low reliability r_L and sets a wholesale price of $\frac{c_L}{1-\eta}$. Each buyer single sources d from the certified high reliability manufacturer.

Lemma A.1 shows when reliability is sufficiently costly ($a \geq a'$), neither manufacturer participates in certification nor improves reliability, and the equilibrium outcomes, including manufacturers' reliability and pricing decisions and buyers' sourcing decisions, are the same as those in the case of $a \geq \bar{\bar{a}}$ under our main model (see Lemma 3). On the other hand, when certification is less costly ($a < a'$), one manufacturer chooses r_H, and the equilibrium outcomes align with the case of $a < \bar{\bar{a}}$ under our main model.

Hereafter we focus on the case where $a < a'$, since neither manufacturer participates in certification nor improves reliability (and thus there is no impact of certification) when $a \geq a'$. Let $\bar{r}_H$ be the threshold in Proposition 1. The following proposition characterizes the impact of certification on patients.

Proposition A.1. *When manufacturers' certification decisions are endogenous, certification leads to a strictly higher expected shortage cost for patients if and only if $r_H < \bar{r}_H$.*

Proposition A.1 shows that our result in Proposition 1 continue to hold under endogenous certification decisions by manufacturers. That is, certification can increase the shortage cost for patients compared to the no certification case if the certification threshold for high reliability is not sufficiently high (i.e., r_H is low). The intuition is similar as before: While certification can motivate manufacturers to improve their reliability, it can also incentivize buyers to pursue less resilient sourcing strategies, resulting greater shortage costs for patients if the manufacturers' reliability improvement is only modest.

Let $\bar{r}_H$, $\bar{a}$, and $\bar{c}_L$ be the thresholds used in Corollary 1. The following proposition characterizes a sufficient condition under which certification leads to a Pareto improvement for all stakeholders.

Proposition A.2. *When manufacturers' certification decisions are endogenous, certification leads to a Pareto improvement for patients, manufacturers, buyers, and the GPO if* $r_H \geq \bar{r}_H$, $a \leq \bar{a} + \frac{f}{r_H(r_H - r_L)D}$ *and* $c_L \geq \bar{c}_L$.

Consistent with Corollary 1, Proposition A.2 shows that certification benefits all stakeholders when (1) the certification threshold for high reliability (i.e., r_H) is sufficiently high; (2) reliability is not too costly (i.e., a is not too high); and (3) the production cost before reliability improvement (i.e., c_L) is relatively high such that manufacturer profit margins are low. This result again demonstrates that our key insights derived from the main model continue to hold when manufacturers' certification decisions are endogenous.

A.1 Proofs

Proof of Lemma A.1. The proof of this lemma is similar to that of Lemma 3. We again proceed in three steps and focus on highlighting the key differences.

Step 1: First, we analyze the buyers' sourcing decisions. Buyers observe both the certification status (which, in the extension with endogenous certification decisions, has three possibilities: certified high reliability, certified low reliability, and non-certified) and the wholesale price of the two manufacturers. Therefore, our characterization of buyers' sourcing strategies in Lemma 1 continues to hold, except that buyers' beliefs about each manufacturer's reliability choice now depend on the observed certification statuses and wholesale prices.

Step 2: For each certification status profile that may arise in equilibrium, we derive necessary conditions that the corresponding equilibrium reliability levels and prices must satisfy. Before proceeding, we first observe that the certification status of certified low reliability cannot arise in equilibrium for any manufacturer, as it is straightforward to show that it is dominated by non-certified (because, compared to non-certified, choosing certified low reliability incurs a certification cost without gaining anything). Therefore, it is sufficient to consider two certification statues: certified high reliability and non-certified.

(i) Suppose both manufacturers choose non-certified in equilibrium. Then, the equilibrium outcomes must be identical as the no certification case characterized in Lemma 2. That is, we have $r_1 = r_2 = r_L$ and $w_1 = w_2 = \max\{\frac{c_L}{1-\eta}, (1 - r_L)p + r_L s\}$ in equilibrium.

(ii) Suppose both manufacturers choose certified high reliability in equilibrium. Then, the equilibrium outcomes must be identical to the case (ii) in Step 2 of the proof of Lemma 3. That is, we have $r_1 = r_2 = r_H$ and $w_1 = w_2 = \max\{\frac{c_H}{1-\eta}, (1-r_H)p + r_H s\}$ in equilibrium.

(iii) Suppose one manufacturer chooses certified high reliability while the other chooses non-certified in equilibrium. Then, following a similar analysis as in case (iii) in Step 2 of the proof of Lemma 3, we conclude that:

(a) Suppose $\frac{c_H}{1-\eta} \leq (1-r_L)p + r_L s$ and $\frac{c_L}{1-\eta} \leq (1-r_H)p + r_H s$. Then, we must have $r_1 = r_H, r_2 = r_L$ and $w_1 = (1-r_L)p + r_L s$, $w_2 = (1-r_H)p + r_H s$ in equilibrium.

(b) Suppose (1) $\frac{c_H}{1-\eta} > (1-r_L)p + r_L s$ or $\frac{c_L}{1-\eta} \leq (1-r_H)p + r_H s$, and (2) $\frac{r_H c_H - r_L c_L}{1-\eta} \geq (r_H - r_L)p$. Then, there must be no equilibrium where one manufacturer chooses certified high reliability while the other chooses non-certified.

(c) Suppose (1) $\frac{c_H}{1-\eta} > (1-r_L)p + r_L s$ or $\frac{c_L}{1-\eta} \leq (1-r_H)p + r_H s$, and (2) $\frac{r_H c_H - r_L c_L}{1-\eta} < (r_H - r_L)p$. Then, we must have $r_1 = r_H, r_2 = r_L$ and $w_1 = \frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} - \epsilon, w_2 = \frac{c_L}{1-\eta}$ in equilibrium.

Step 3: Finally, we characterize the conditions under which each certification status profile arises in equilibrium. Let Π_i^{AB} denote the expected profit of manufacturer i given the certification status A of manufacturer 1 and B of manufacturer 2, where $A, B \in \{C \text{ (certified high reliability)}, N \text{ (non-certified)}\}$ and the corresponding reliability and pricing choices satisfy the equilibrium conditions characterized in Step 2.

(i) We first compare Π_1^{NN} with Π_1^{CN}. Consider the following three cases:

(a) Suppose $\frac{c_H}{1-\eta} \leq (1-r_L)p + r_L s$, $\frac{c_L}{1-\eta} \leq (1-r_H)p + r_H s$. Then, following a similar analysis as in case (i.a) of Step 3 in the proof of Lemma 3 and further considering the cost of participating in certification, we have $\Pi_1^{NN} > \Pi_1^{CN}$, and thus the certification status CN cannot be sustained in equilibrium.

(b) Suppose (1) $\frac{c_H}{1-\eta} > (1-r_L)p + r_L s$ or $\frac{c_L}{1-\eta} > (1-r_H)p + r_H s$, and (2) $\frac{r_H c_H - r_L c_L}{1-\eta} \geq (r_H - r_L)p$. Then, following a similar analysis as in case (i.b) of Step 3 in the proof of Lemma 3 and further considering the cost of participating in certification, we conclude that the certification status CN cannot be sustained in equilibrium.

(c) Suppose (1) $\frac{c_H}{1-\eta} > (1-r_L)p + r_L s$ or $\frac{c_L}{1-\eta} > (1-r_H)p + r_H s$, and (2) $\frac{r_H c_H - r_L c_L}{1-\eta} < (r_H - r_L)p$. Then, following a similar analysis as in case (i.c) of Step 3 in the proof of Lemma 3 and further considering the cost of participating in certification, we have the following:

When $\frac{c_L}{1-\eta} > (1-r_L)p + r_L s$, we have

$$\begin{aligned}\Pi_1^{NN} - \Pi_1^{CN} &= 0 - (1-\eta)r_H\left(\frac{r_L\frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} - \frac{c_H}{1-\eta}\right)D + f \\ &= (1-\eta)\left(\frac{r_H c_H - r_L c_L}{1-\eta} - (r_H - r_L)p + \frac{f}{(1-\eta)D}\right)D.\end{aligned}$$

Then, we have $\Pi_1^{NN}(r_L) < \Pi_1^{CN}(r_H)$ if $\frac{r_H c_H - r_L c_L}{1-\eta} < (r_H - r_L)p - \frac{f}{(1-\eta)D}$.

When $\frac{c_L}{1-\eta} \leq (1-r_L)p + r_L s$, we have

$$\begin{aligned}&\Pi_1^{NN}(r_L) - \Pi_1^{CN}(r_H) \\ =&(1-\eta)r_L\left((1-r_L)p + r_L s - \frac{c_L}{1-\eta}\right)D - (1-\eta)r_H\left(\frac{r_L\frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} - \frac{c_H}{1-\eta}\right)D + f \\ =&(1-\eta)\left(\frac{r_H c_H - r_L c_L}{1-\eta} - (r_H - r_L)p + \frac{f}{(1-\eta)D} - r_L\left(\frac{c_L}{1-\eta} - ((1-r_L)p + r_L s)\right)\right)D.\end{aligned}$$

Then, we have $\Pi_1^{NN}(r_L) < \Pi_1^{CN}(r_H)$ if $\frac{r_H c_H - r_L c_L}{1-\eta} < (r_H - r_L)p - \frac{f}{(1-\eta)D} + r_L(\frac{c_L}{1-\eta} - ((1-r_L)p + r_L s))$.

Then, following a similar analysis as in case (i.c) of Step 3 in the proof of Lemma 3, we have $\Pi_1^{NN}(r_L) < \Pi_1^{CN}(r_H)$ when (1) $\frac{c_H}{1-\eta} > (1-r_L)p + r_L s$ or $\frac{c_L}{1-\eta} > (1-r_H)p + r_H s$, and (2) $\frac{r_H c_H - r_L c_L}{1-\eta} < (r_H - r_L)p - \frac{f}{(1-\eta)D} - r_L((1-r_L)p + r_L s - \frac{c_L}{1-\eta})^+$. Otherwise, under condition (c), we have $\Pi_1^{NN}(r_L) \geq \Pi_1^{CN}(r_H)$. Moreover, following a similar analysis as before, we have that condition (1) is redundant and conditions (2) is equivalent to $a < a' := \frac{1}{r_H - r_L}\frac{(r_H - r_L)((1-\eta)p - c_L) - \frac{f}{D}}{r_H} - (1-\eta)\frac{r_L}{r_H}\left((1-r_L)p + r_L s - \frac{c_L}{1-\eta}\right)^+$.

(ii) Following a similar analysis as in case (ii) of Step 3 in the proof of Lemma 3 and further considering the certification cost of choosing certified high reliability, we conclude that the certification status CC cannot be sustained in equilibrium.

Combining all cases, we conclude that when $a < a'$, one manufacturer participates in certification can chooses a reliability level of r_H while the other remains non-certified and chooses a reliability level of r_L in equilibrium. Otherwise, neither manufacturer participates in certification nor improves reliability. □

Proof of Proposition A.1. Recall that we focus on $a < a'$. Based on Lemma A.1, we know that one manufacturer chooses r_H while the other manufacturer chooses r_L; buyers source a total of D from the high reliability manufacturer while nothing from the low-reliability manufacturer.

Thus, the shortage quantity is exactly the same as that under the main model in the case of $a < \bar{\bar{a}}$ (see Lemma 3). Then, certification leads to a strictly higher expected shortage cost for patients if and only if $r_H < \bar{r}_H$. □

Proof of Proposition A.2. Recall that we focus on $a < a'$. Based on Lemma A.1, we know that with endogenous certification decisions, the reliability, pricing, and sourcing decisions in equilibrium are the same as those under our main model in the case of $a < \bar{\bar{a}}$ (see Lemma 3). Thus, certification leads to an equivalent outcome on each performance measure as presented in Proposition 1-4, except that the low-reliability manufacturer will benefit from saving the certification cost f.

Recall from our proof of Proposition 2 that when both manufacturers are certified but only one chooses r_H, certification leads to a lower expected manufacturer profit if and only if

$$c_H \geq \frac{r_L c_L + (1-\eta)(r_H - r_L)p - \frac{2f}{D} - 2r_L(\bar{\bar{c}}_L(r_L) - c_L)^+}{r_H} \iff a \geq \bar{a}.$$

Following a similar analysis, we conclude that if only the high-reliability manufacturer gets certified and incurs the certification cost, certification leads to a lower expected manufacturer profit if and only if

$$c_H \geq \frac{r_L c_L + (1-\eta)(r_H - r_L)p - \frac{f}{D} - 2r_L(\bar{\bar{c}}_L(r_L) - c_L)^+}{r_H} \iff a \geq \bar{a} + \frac{f}{r_H(r_H - r_L)D}.$$

Then, following a similar analysis as in the proof of Corollary 1, we conclude that certification leads to a Pareto improvement if $r_H \geq \bar{r}_H$, $a \leq \bar{a} + \frac{f}{r_H(r_H-r_L)D}$ and $c_L \geq \bar{c}_L$. □

B Convex Shortage Cost for Buyers

In this section, we extend our model to consider convex shortage costs for buyers. Specifically, we now consider an additional quadratic term in each buyer's shortage cost to capture the potentially increasing marginal cost for managing shortages (e.g., buyers may need to procure more expensive alternatives when the shortage quantity is higher). That is, the expected cost for buyer j defined in Equation 2 is revised as follows:

$$\begin{aligned}
& g((r_1, r_2), (w_1, w_2), (Q_{1j}, Q_{2j})) \\
= \; & r_1 r_2 \left(p(d - Q_{1j} - Q_{2j})^+ + p'((d - Q_{1j} - Q_{2j})^+)^2 + w_1 Q_{1j} + w_2 Q_{2j} - s(Q_{1j} + Q_{2j} - d)^+ \right) \\
& + r_1(1 - r_2) \left(p(d - Q_{1j})^+ + p'((d - Q_{1j})^+)^2 + w_1 Q_{1j} - s(Q_{1j} - d)^+ \right) \\
& + (1 - r_1) r_2 \left(p(d - Q_{2j})^+ + p'((d - Q_{2j})^+)^2 + w_2 Q_{2j} - s(Q_{2j} - d)^+ \right) \\
& + (1 - r_1)(1 - r_2) \left(pd + p'd^2 \right),
\end{aligned} \tag{B.1}$$

where $p' \geq 0$.

As shown in Lemma 2, without certification, when the reliability improvement cost exceeds a threshold (i.e., Assumption 1), manufacturers retain the baseline reliability level r_L. With certification, under the same conditions, manufacturers either choose the baseline reliability r_L or the certification threshold r_H for high reliability (see Lemma 3). Motivated by this observation and for simplicity, in this section, we focus on the scenario where each manufacturer chooses between two discrete reliability levels r_L and r_H. Further, in line with our main text, we focus on scenarios where the reliability improvement cost is high such that no manufacturer chooses r_H under no certification.

Assumption B.1.

$$a \geq \max\{(1-\eta)\frac{-(r_H - r_L)\frac{c_L}{1-\eta} + 2\frac{r_H + r_L}{r_H - r_L}(1 - r_L)p'd + p}{r_H^2}, (1-\eta)\frac{4(1 - r_L)p'd}{r_L}\}.$$

Under this assumption, similar to before, it can be shown that while neither manufacturer improves their reliability without certification, certification can effectively incentivize reliability improvement. Moreover, it can also be shown that when the certification threshold for high reliability (i.e., r_H) is not high enough, certification may strictly increase patients' shortage cost once manufacturers adjust their prices and buyers adjust their sourcing strategies (see the proof of Proposition B.1 for formal characterizations). Finally, the following proposition shows that certification leads to a Pareto improvement under similar conditions as in our main model. We prove this proposition by considering both convex costs for buyers and endogenous certification decisions for manufacturers (as considered in Supplemental Appendix A) to demonstrate that our key insights remain valid when relaxing both assumptions of linear buyer costs and exogenous certification decisions.

Proposition B.1. *When each buyer's cost is defined by Equation B.1 and manufacturers' certification decisions are endogenous, there exist thresholds $\widehat{r}_H$, $\widehat{a}$, and $\widehat{c}_L$ such that certification leads to a Pareto improvement for patients, manufacturers, buyers, and the GPO if $r_H \geq \widehat{r}_H$, $a \leq \widehat{a}$, and $c_L \geq \widehat{c}_L$.*

Consistent with Corollary 1, Proposition B.1 shows that certification benefits all stakeholders when (1) the certification threshold for high reliability (i.e., r_H) is sufficiently high; (2) reliability is not too costly (i.e., a is not too high); and (3) the production cost before reliability improvement (i.e., c_L) is relatively high such that manufacturer profit margins are low. This result demonstrates that our insights on when reliability certification can benefit all stakeholders continue to hold under both convex shortage costs for buyers and endogenous certification decisions by manufacturers.

B.1 Proofs

Proof of Proposition B.1. Without loss of generality, we prove this proposition by considering $p' > 0$. Let

$$\begin{aligned}
\widehat{c}_L &= (1-\eta)\left((1-r_L)(2p'd+p)+r_L s\right),\\
\widehat{c}_H &= \min\{\frac{r_L c_L - (1-\eta)\left(2(r_H+2r_L-3r_H r_L)p'd-(r_H-r_L)p\right)}{r_H},\\
&\qquad \frac{r_L c_L - (1-\eta)\left(2r_L(2-r_H-r_L)p'd-(r_H-r_L)p\right)-\frac{f}{D}}{r_H}\},\\
\widehat{r}_H &= \max\{\frac{1+\sqrt{1-\frac{4f}{(1-\eta)p'dD}}}{2}\mathbf{1}[1>\frac{4f}{(1-\eta)p'dD}], \frac{3-r_L}{2}r_L, \frac{p+2(2-r_L)p'd}{p+2r_L p'd}r_L\},\\
\widehat{a} &= \frac{\widehat{c}_H - c_L}{r_H-r_L}.
\end{aligned}$$

Let $c_H = a(r_H-r_L)+c_L$. Then, we have $a \leq \widehat{a} \iff c_H \leq \widehat{c}_H$. Let $\bar{w} = \frac{r_H c_H - r_L c_L}{(1-\eta)(r_H-r_L)}$. Then, by Assumption B.1, we have

$$a \geq \max\{(1-\eta)\frac{-(r_H-r_L)\frac{c_L}{1-\eta}+2\frac{r_H+r_L}{r_H-r_L}(1-r_L)p'd+p}{r_H^2}, (1-\eta)\frac{4(1-r_L)p'd}{r_L}\}$$

$$\Longleftrightarrow \bar{w} \geq \max\{\frac{r_L \frac{c_L}{1-\eta} + 2(r_H + r_L)(1-r_L)p'd + (r_H - r_L)p}{r_H}, \frac{c_H}{1-\eta} + 4(1-r_L)p'd\}.$$

We next prove the proposition in the following three steps:

Step 1: We characterize the equilibrium decisions without certification.

First, we characterize the optimal sourcing quantities Q^*_{1j}, Q^*_{2j} for each buyer $j \in \{1, ..., N\}$ under given wholesale prices w_1, w_2. Since all players' decisions must be sequentially rational in equilibrium, and following a similar analysis as in the proof of Lemma 2, in equilibrium, each manufacturer $i \in \{1, 2\}$ chooses a reliability level r_i and a wholesale price w_i such that $r_i(w_i) = r_H$ if $w_i > \bar{w}$ and $r_i(w_i) = r_L$ otherwise. Each buyer's belief about a manufacturer's reliability choice must be consistent with this relationship.

Following the same arguments as in the proof of Lemma 1, we know that the optimal sourcing quantities must satisfy $Q^*_{ij} \leq d$ and $Q^*_{1j} + Q^*_{2j} \geq d$. We next show that given $c_L \geq \widehat{c}_L = (1-\eta)((1-r_L)(2p'd+p)+r_L s)$, the optimal sourcing quantities of each buyer j must satisfy $Q^*_{1j} + Q^*_{2j} = d$. We prove this by using contradiction: Suppose $Q_{ij} \leq d, i \in \{1, 2\}$ and $Q_{1j} + Q_{2j} > d$. When we decrease Q_{1j} by a small quantity ϵ (i.e., $Q_{1j} - \epsilon > 0$ and $Q_{1j} + Q_{2j} - \epsilon > d$), the expected buyer's cost would decrease by $r_1(w_1)w_1\epsilon - r_1(w_1)(1 - r_2(w_2))\left(p'\left(2(d - Q_{1j})\epsilon + \epsilon^2\right) + p\epsilon\right) - r_1(w_1)r_2(w_2)s\epsilon$. Since $w_1 \geq \frac{c_L}{1-\eta}$ and $2Q_{1j} - \epsilon > 0$, we have

$$\begin{aligned}
& w_1 - (1 - r_2(w_2))\left(p'\left(2(d - Q_{1j}) + \epsilon\right) + p\right) - r_2(w_2)s \\
\geq & \frac{c_L}{1-\eta} - (1 - r_2(w_2))\left(p'\left(2(d - Q_{1j}) + \epsilon\right) + p\right) - r_2(w_2)s \\
= & \frac{c_L}{1-\eta} - (1 - r_2(w_2))\left((2p'd + p) - p'(2Q_{1j} - \epsilon)\right) - r_2(w_2)s \\
> & \frac{c_L}{1-\eta} - (1 - r_2(w_2))\left(2p'd + p\right) - r_2(w_2)s \\
> & \frac{c_L}{1-\eta} - (1 - r_L)\left(2p'd + p\right) - r_L s \\
\geq & 0,
\end{aligned}$$

where the last inequality follows from $c_L \geq (1-\eta)\left((1-r_L)(2p'd+p) + r_L s\right)$.

Thus, we can simplify buyer j's objective function in Equation B.1 by replacing $(Q_{1j} - d)^+$, $(Q_{2j} - d)^+$, $(d - Q_{1j} - Q_{2j})^+$ and $(Q_{1j} + Q_{2j} - d)^+$ by 0. Further, $d - Q_{1j}, d - Q_{2j}$ are non-negative. As a result, buyer j's expected cost can be simplified as follows:

$$\begin{aligned}
& r_1(w_1)r_2(w_2)\left(w_1 Q_{1j} + w_2 Q_{2j}\right) \\
& + r_1(w_1)(1 - r_2(w_2))\left(p'(d - Q_{1j})^2 + p(d - Q_{1j}) + w_1 Q_{1j}\right) \\
& + (1 - r_1(w_1))r_2(w_2)\left(p'(d - Q_{2j})^2 + p(d - Q_{2j}) + w_2 Q_{2j}\right) \\
& + (1 - r_1(w_1))(1 - r_2(w_2))\left(p'd^2 + pd\right).
\end{aligned}$$

Replacing Q_{2j} by $d - Q_{1j}$, each buyer's expected cost can be written as a convex function of Q_{1j}. By applying the first-order condition on Q_{1j}, we obtain

$$\begin{aligned}
& r_1(w_1)r_2(w_2)(w_1 - w_2) + r_1(w_1)(1 - r_2(w_2))(2p'(Q_{1j} - d) - p + w_1) + (1 - r_1(w_1))r_2(w_2)(2p'Q_{1j} + p - w_2) \\
= & 2\left(r_1(w_1) + r_2(w_2) - 2r_1(w_1)r_2(w_2)\right)p'Q_{1j} - r_2(w_2)w_2 + r_1(w_1)w_1 - 2r_1(w_1)(1 - r_2(w_2)p'd + (r_2(w_2) - r_1(w_1))p
\end{aligned}$$

$=0.$

Then, the quantity Q_{1j} that satisfies the first-order condition is given by

$$Q_{1j} = \frac{r_2(w_2)w_2 - r_1(w_1)w_1 + 2r_1(w_1)(1-r_2(w_2))p'd + (r_1(w_1) - r_2(w_2))p}{2\left(r_1(w_1) + r_2(w_2) - 2r_1(w_1)r_2(w_2)\right)p'}.$$

Further considering the constraint that $Q_{1j} \in [0, d]$, buyer j's optimal sourcing quantities are given as follows:

$$Q^*_{1j}(w_1, w_2) = \begin{cases} 0 \quad \text{if } w_1 \geq \dfrac{r_2(w_2)w_2 + 2r_1(w_1)(1-r_2(w_2))p'd + (r_1(w_1) - r_2(w_2))p}{r_1(w_1)}, \\ d \quad \text{if } w_1 \leq \dfrac{r_2(w_2)w_2 - 2r_2(w_2)(1-r_1(w_1))p'd + (r_1(w_1) - r_2(w_2))p}{r_1(w_1)}, \\ \dfrac{r_2(w_2)w_2 - r_1(w_1)w_1 + 2r_1(w_1)(1-r_2(w_2))p'd + (r_1(w_1) - r_2(w_2))p}{2\left(r_1(w_1) + r_2(w_2) - 2r_1(w_1)r_2(w_2)\right)p'} \quad \text{otherwise,} \end{cases} \tag{B.2}$$

and $Q^*_{2j}(w_1, w_2) = d - Q^*_{1j}(w_1, w_2)$.

Second, we characterize each manufacturer's reliability and pricing decisions. As discussed above, in equilibrium, each manufacturer $i \in \{1, 2\}$ chooses a reliability level r_i and a wholesale price w_i such that $r_i(w_i) = r_H$ if $w_i > \bar{w}$ and $r_i(w_i) = r_L$ otherwise. Therefore, as before, to characterize the equilibrium, it is sufficient to consider each manufacturer's reliability r_i as a function of its wholesale price w_i and focus on analyzing each manufacturer's pricing decision. In order to derive the equilibrium prices, we consider the following two possible scenarios:

(a) Suppose we have $Q^*_{ij} = 0$ and $Q^*_{(-i)j} = d$ in equilibrium. Then, manufacturer i must have set its price at $\frac{c_L}{1-\eta}$ (which implies $r_i = r_L$) while the other manufacturer $-i$ sets the highest price that allows it to receive d from each buyer. That is, $w_i = \frac{c_L}{1-\eta}, w_{-i} = \frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_{-i}(w_{-i}))p'd + (r_{-i}(w_{-i}) - r_L)p}{r_{-i}(w_{-i})}$.

(b) Suppose we have $Q^*_{ij} > 0$ for $i \in \{1, 2\}$ in equilibrium. Based on Equation B.2, for $i \in \{1, 2\}$, we have

$$\frac{r_{-i}(w_{-i})w_{-i} - 2r_{-i}(w_{-i})(1-r_i(w_i))p'd + (r_i(w_i) - r_{-i}(w_{-i}))p}{r_i(w_i)} < w_i < \frac{r_{-i}(w_{-i})w_{-i} + 2r_i(w_i)(1-r_{-i}(w_{-i}))p'd + (r_i(w_i) - r_{-i}(w_{-i}))p}{r_i(w_i)}. \tag{B.3}$$

For any w_1, w_2 that satisfy this condition, manufacturer i's expected profit (Equation 1) is given as follows, where $Q^*_{ij}(w_1, w_2)$ is characterized in Equation B.2:

$$\begin{aligned} &(1-\eta)r_i(w_i)\left(w_i - \frac{c_i(r_i(w_i))}{1-\eta}\right)Q^*_{ij}(w_1, w_2) \\ =&(1-\eta)r_i(w_i)\left(w_i - \frac{c_i(r_i(w_i))}{1-\eta}\right)\frac{r_{-i}(w_{-i})w_{-i} - r_i(w_i)w_i + 2r_i(w_i)(1-r_{-i}(w_{-i}))p'd + (r_i(w_i) - r_{-i}(w_{-i}))p}{2(r_1(w_1) + r_2(w_2) - 2r_1(w_1)r_2(w_2))p'}. \end{aligned} \tag{B.4}$$

Let w^*_1, w^*_2 denote the equilibrium prices. Then, for $i \in \{1, 2\}$, w^*_i must be the best response of manufacturer i when the wholesale price of manufacturer $-i$ is fixed at w^*_{-i}.

(i) Suppose $w^*_i > \bar{w}$. Then, we have $r_i(w^*_i) = r_H$ and $c_i(r_i(w^*_i)) = c_H$.

- Suppose the equilibrium prices satisfy condition (a) above. Then, we must have

$$w^*_i = \frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H}, w^*_{-i} = \frac{c_L}{1-\eta}.$$

Under Assumption B.1, we have

$$\bar{w} \geq \frac{r_L \frac{c_L}{1-\eta} + 2(r_H + r_L)(1-r_L)p'd + (r_H - r_L)p}{r_H} > \frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H} = w_i^*,$$

which contradicts with $w_i^* > \bar{w}$.

- Suppose the equilibrium prices satisfy condition (b) above. Then, manufacturer i's expected profit is given by Equation B.4. Since $w_i^* > \bar{w}$, w_i^* must satisfy the first-order condition:

$$w_i^* = \frac{r_{-i}(w_{-i}^*)w_{-i}^* + r_H\left(\frac{c_H}{1-\eta} + 2\left(1 - r_{-i}(w_{-i}^*)\right)p'd\right) + \left(r_H - r_{-i}(w_{-i}^*)\right)p}{2r_H}. \tag{B.5}$$

(ii) Suppose $w_i^* < \bar{w}$. Then, we have $r_i(w_i^*) = r_L$ and $c_i(r_i(w_i^*)) = c_L$.

- Suppose the equilibrium prices satisfy condition (a) above. Then:
  (1) If $w_{-i}^* \leq \bar{w}$ (which implies $r_{-i}(w_{-i}^*) = r_L$), then the manufacturer who receives d from each buyer must have its price at $\frac{c_L}{1-\eta} - 2(1-r_L)p'd$. This contradicts with $w_1, w_2 \geq \frac{c_L}{1-\eta}$.
  (2) If $w_{-i}^* > \bar{w}$ (which implies $r_{-i}(w_{-i}^*) = r_H$), then following a similar analysis as in case (i), we can reach a contradiction.
- Suppose the equilibrium prices satisfy condition (b) above. Then, manufacturer i's expected profit is given by Equation B.4. Since $w_i^* < \bar{w}$, w_i^* must satisfy the first-order condition:

$$w_i^* = \frac{r_{-i}(w_{-i}^*)w_{-i}^* + r_L\left(\frac{c_L}{1-\eta} + 2\left(1 - r_{-i}(w_{-i}^*)\right)p'd\right) + \left(r_L - r_{-i}(w_{-i}^*)\right)p}{2r_L}. \tag{B.6}$$

(iii) Suppose $w_i^* = \bar{w}$. Then, we know that $r_i(w_i^*) = r_L$ and $c_i(r_i(w_i^*)) = c_L$. Following a similar analysis as in case (ii), we know that there must be no equilibrium prices that satisfy condition (a). Thus, it is sufficient to focus on condition (b). Further, for $w_i \leq \bar{w}$, manufacturer i's expected profit is a concave quadratic function of w_i. Then, given $w_i^* = \bar{w}$, we must have

$$\frac{r_{-i}(w_{-i}^*)w_{-i}^* + r_L\left(\frac{c_L}{1-\eta} + 2(1 - r_{-i}(w_{-i}^*))p'd\right) + \left(r_L - r_{-i}(w_{-i}^*)\right)p}{2r_L} \geq \bar{w}.$$

We divide our analysis into the following two cases:

(1) If $w_{-i}^* \leq \bar{w}$ (which implies $r_{-i}(w_{-i}^*) = r_L$), then we have

$$\frac{r_L w_{-i}^* + r_L\left(\frac{c_L}{1-\eta} + 2(1-r_L)p'd\right)}{2r_L} \geq \bar{w}$$
$$\Longleftrightarrow w_{-i}^* \geq \frac{2r_L\bar{w} - r_L\left(\frac{c_L}{1-\eta} + 2(1-r_L)p'd\right)}{r_L} = 2\bar{w} - \left(\frac{c_L}{1-\eta} + 2(1-r_L)p'd\right)$$
$$\Longrightarrow w_{-i}^* > \bar{w},$$

where the last inequality follows from Assumption B.1: $\bar{w} \geq \frac{c_H}{1-\eta} + 4(1-r_L)p'd > \frac{c_L}{1-\eta} + 2(1-r_L)p'd$. Thus, this leads to a contradiction.

(2) If $w^*_{-i} > \bar{w}$ (which implies $r_{-i}(w^*_{-i}) = r_H$), then we have

$$\frac{r_H w^*_{-i} + r_L\left(\frac{c_L}{1-\eta} + 2(1-r_H)p'd\right) + (r_L - r_H)p}{2r_L} \geq \bar{w}$$

$$\Longleftrightarrow w^*_{-i} \geq \frac{2r_L\bar{w} - r_L\left(\frac{c_L}{1-\eta} + 2(1-r_H)p'd\right) + (r_H - r_L)p}{r_H},$$

$$\Longleftrightarrow w^*_{-i} \geq \frac{r_L\bar{w} + 2r_H(1-r_L)p'd + (r_H - r_L)p}{r_H} + \frac{r_L\bar{w} - r_L\frac{c_L}{1-\eta} - 2(r_H + r_L - 2r_Hr_L)p'd}{r_H}.$$

Under Assumption B.1, we have $\bar{w} \geq \frac{c_H}{1-\eta} + 4(1-r_L)p'd$. Then, we know that

$$w^*_{-i} \geq \frac{r_L\bar{w} + 2r_H(1-r_L)p'd + (r_H - r_L)p}{r_H} + \frac{r_L\frac{c_H - c_L}{1-\eta} + 4r_L(1-r_L)p'd - 2(r_H + r_L - 2r_Hr_L)p'd}{r_H}.$$

Given $\bar{w} \geq \frac{c_H}{1-\eta} + 4(1-r_L)p'd$, we also have $r_L\frac{c_H - c_L}{1-\eta} \geq 4(r_H - r_L)(1-r_L)p'd$. Then,

$$\begin{aligned} w^*_{-i} &\geq \frac{r_L\bar{w} + 2r_H(1-r_L)p'd + (r_H - r_L)p}{r_H} + \frac{4r_H(1-r_L)p'd - 2(r_H + r_L - 2r_Hr_L)p'd}{r_H} \\ &= \frac{r_L\bar{w} + 2r_H(1-r_L)p'd + (r_H - r_L)p}{r_H} + \frac{2(r_H - r_L)p'd}{r_H} \\ &> \frac{r_L\bar{w} + 2r_H(1-r_L)p'd + (r_H - r_L)p}{r_H}. \end{aligned}$$

Then, based on condition B.3, manufacturer $-i$ will receive zero demand, and thus it will have the incentive to deviate to a lower price to obtain a strictly positive profit.

Therefore, we must have $w^*_i \neq \bar{w}$ in equilibrium.

With these observations, we conclude that in equilibrium, we must have $Q^*_{ij} > 0$ for $i \in \{1, 2\}$ in equilibrium, and the equilibrium price w^*_i must satisfy either Equation B.5 or Equation B.6. We next prove that we must have $w^*_1 = w^*_2 = \frac{c_L}{1-\eta} + 2(1-r_L)p'd$ in equilibrium without certification.

(i) Suppose $w^*_1 > \bar{w}, w^*_2 > \bar{w}$ (which implies $r_1 = r_2 = r_H$). Then, by solving Equation B.5 for $i = 1, 2$, we obtain $w^*_1 = w^*_2 = \frac{c_H}{1-\eta} + 2(1-r_H)p'd$. Based on Assumption B.1, we have $\bar{w} \geq \frac{c_H}{1-\eta} + 4(1-r_L)p'd \geq \frac{c_H}{1-\eta} + 2(1-r_H)p'd = w^*_1$, which contradicts with $w^*_1 > \bar{w}$. Therefore, there must be no equilibrium prices that satisfy $w^*_1 > \bar{w}, w^*_2 > \bar{w}$.

(ii) Suppose $w^*_1 > \bar{w}, w^*_2 < \bar{w}$ (which implies $r_1 = r_H, r_2 = r_L$). Then, by Equation B.5, we have $w^*_1 = \frac{r_L w^*_2 + r_H(\frac{c_H}{1-\eta} + 2(1-r_L)p'd) + (r_H - r_L)p}{2r_H}$. Moreover, w^*_1 must satisfy B.3, which implies

$$\begin{aligned} &\frac{r_L w^*_2 + r_H(\frac{c_H}{1-\eta} + 2(1-r_L)p'd) + (r_H - r_L)p}{2r_H} - \frac{r_L w^*_2 - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H} \\ =&\frac{-r_L w^*_2 + r_H\frac{c_H}{1-\eta} + 2(r_H + 2r_L - 3r_Hr_L)p'd - (r_H - r_L)p}{2r_H} \\ >&0. \end{aligned}$$

This implies that $w^*_2 < \frac{r_H\frac{c_H}{1-\eta} + 2(r_H + 2r_L - 3r_Hr_L)p'd - (r_H - r_L)p}{r_L} \leq \frac{c_L}{1-\eta}$, where the last inequality follows from $c_H \leq \hat{c}_H$, i.e., $\frac{r_Hc_H - r_Lc_L}{1-\eta} \leq -2(r_H + 2r_L - 3r_Hr_L)p'd + (r_H - r_L)p$. This contradicts with

$w_2 \geq \frac{c_L}{1-\eta}$, which is needed to ensure a nonnegative profit of manufacturer 2. Therefore, there must be no equilibrium prices that satisfy $w_1^* > \bar{w}, w_2^* < \bar{w}$.

(iii) Suppose $w_1^* < \bar{w}, w_2^* > \bar{w}$ (which implies $r_1 = r_L, r_2 = r_H$). Then, we can reach a contradiction by following a similar analysis as in the previous case.

(iv) Suppose $w_1^* < \bar{w}, w_2^* < \bar{w}$ (which implies $r_1 = r_2 = r_L$). Then, by solving Equation B.6 for $i = 1, 2$, we obtain $w_1^* = w_2^* = \frac{c_L}{1-\eta} + 2(1 - r_L)p'd$. We next prove that these are equilibrium prices.

First, under Assumption B.1, we have $\bar{w} \geq \frac{c_H}{1-\eta} + 4(1 - r_L)p'd > \frac{c_L}{1-\eta} + 2(1 - r_L)p'd$. That is, the conditions $w_1^* < \bar{w}, w_2^* < \bar{w}$ are satisfied.

Second, given $w_1^* = w_2^* = \frac{c_L}{1-\eta} + 2(1 - r_L)p'd < \bar{w}$, for $i \in \{1, 2\}$, we know that

$$\frac{r_{-i}(w_{-i})w_{-i} - 2r_{-i}(w_{-i})(1 - r_i(w_i))p'd + (r_i(w_i) - r_{-i}(w_{-i}))\, p}{r_i(w_i)} = \frac{c_L}{1-\eta},$$

$$\frac{r_{-i}(w_{-i})w_{-i} + 2r_i(w_i)(1 - r_{-i}(w_{-i}))p'd + (r_i(w_i) - r_{-i}(w_{-i}))\, p}{r_i(w_i)} = \frac{c_L}{1-\eta} + 4(1 - r_L)p'd.$$

Thus, $w_1^*, w_2^* = \frac{c_L}{1-\eta} + 2(1 - r_L)p'd$ satisfy B.3, i.e., the condition that each manufacturer receives a positive demand.

It remains to prove that given $w_2 = \frac{c_L}{1-\eta} + 2(1 - r_L)p'd$, manufacturer 1 does not have an incentive to deviate from $w_1 = \frac{c_L}{1-\eta} + 2(1 - r_L)p'd$.

(i) Since $w_1^* = w_2^* = \frac{c_L}{1-\eta} + 2(1 - r_L)p'd$ are obtained from Equation B.6, and from the above analysis we have $\frac{c_L}{1-\eta} < w_1^* < \frac{c_L}{1-\eta} + 4(1 - r_L)p'd < \bar{w}$ (where the last inequality holds due to Assumption B.1), it follows that given $w_2 = \frac{c_L}{1-\eta} + 2(1 - r_L)p'd$, $w_1 = \frac{c_L}{1-\eta} + 2(1 - r_L)p'd$ is the best response among all w_1 that satisfies $\frac{c_L}{1-\eta} < w_i < \frac{c_L}{1-\eta} + 4(1 - r_L)p'd$.

(ii) Deviating to $w_1 = \frac{c_L}{1-\eta}$ leads to a zero profit, which is strictly worse than $w_1 = \frac{c_L}{1-\eta} + 2(1 - r_L)p'd$.

(iii) Deviating to a price such that $\frac{c_L}{1-\eta} + 4(1 - r_L)p'd \leq w_1 \leq \bar{w}$ leads to a zero profit due to a zero demand from buyers, which is strictly worse than $w_1 = \frac{c_L}{1-\eta} + 2(1 - r_L)p'd$.

(iv) Deviating to a price such that $w_1 > \bar{w}$ (then $r_1 = r_H$) also leads to a zero profit due to a zero demand from buyers, as it violates the second inequality in B.3. Under Assumption B.1, we have

$$\begin{aligned} w_1 > \bar{w} \geq & \frac{r_L \frac{c_L}{1-\eta} + 2(r_H + r_L)(1 - r_L)p'd + (r_H - r_L)p}{r_H} \\ = & \frac{r_L \left(\frac{c_L}{1-\eta} + 2(1 - r_L)p'd\right) + 2r_H(1 - r_L)p'd + (r_H - r_L)p}{r_H}. \end{aligned}$$

Therefore, $w_1^* = w_2^* = \frac{c_L}{1-\eta} + 2(1 - r_L)p'd$ are equilibrium prices.

Combining our analysis for all cases, we conclude that without certification, in equilibrium, each manufacturer chooses r_L and sets a price of $\frac{c_L}{1-\eta} + 2(1 - r_L)p'd$, and each buyer sources $\frac{d}{2}$ from each manufacturer.

Step 2: We next characterize the equilibrium decisions with certification.

First, we analyze the buyers' sourcing decisions. With certification, buyers observe both the reliability level and the wholesale price of the manufacturers who get certified. Then, each buyer's sourcing strategy follows from Equation B.2 in Step 1 except that $r_i(w_i)$ should be replaced with the observed reliability level r_i if manufacturer i gets certified.

Second, we characterize the manufacturers' pricing decisions. Recall that we consider endogenous certification decisions. Then, when manufacturers make pricing decisions, each manufacturer can observe whether the other manufacturer chooses certified high reliability or does not get certified (following a similar analysis as in the proof of Lemma A.1, it can be shown that certified low reliability is dominated by non-certified low reliability). Consider the following three cases:

Suppose neither manufacturer gets certified. Then, the subsequent equilibrium outcomes are equivalent to those under no certification in Step 1. That is, both manufacturers chooses r_L receives a demand $\frac{d}{2}$ from each buyer at the price of $w_1^* = w_2^* = \frac{c_L}{1-\eta} + 2(1-r_L)p'd$.

Suppose both manufacturers choose certified high reliability. In this case, it is sufficient to consider $w_1, w_2 \geq \frac{c_H}{1-\eta}$. Consider the following two cases. First, suppose $Q_i^* = 0$ and $Q_{-i}^* = d$ in equilibrium, then based on our characterization for buyers' optimal sourcing decisions, we must have $w_i^* = \frac{c_H}{1-\eta}$ and $w_{-i}^* = \frac{c_H}{1-\eta} - 2(1-r_H)p'd < \frac{c_H}{1-\eta}$, which leads to a contradiction. Second, suppose $Q_i^* > 0$ and $Q_{-i}^* > 0$ in equilibrium. Then, by solving Equation B.5 for $i = 1, 2$, we obtain $w_1^* = w_2^* = \frac{c_H}{1-\eta} + 2(1-r_H)p'd$. Following similar arguments as for proving the equilibrium prices in Step 1, we conclude that these are the equilibrium prices.

Suppose one manufacturer (say, manufacturer 1) chooses certified high reliability while the other manufacturer does not get certified. In this case, it is without loss of generality to consider $w_1 \geq \frac{c_H}{1-\eta}$. Further, since manufacturer 2's reliability decision is private information, as discussed in Step 1, in equilibrium, it must choose a reliability level r_2 and a wholesale price w_2 such that $r_2(w_2) = r_H$ if $w_2 > \bar{w}$ and $r_2(w_2) = r_L$ otherwise. Therefore, as before, to characterize the equilibrium, it is sufficient to consider manufacturer 2's reliability r_2 as a function of its wholesale price w_2. In order to derive the equilibrium prices, we consider the following three possible scenarios:

(i) Suppose we have $Q_{1j}^* = 0$ and $Q_{2j}^* = d$ in equilibrium. Then, based on our characterization for buyers' optimal sourcing decisions, the equilibrium prices must be given by $w_1^* = \frac{c_H}{1-\eta}$ and $w_2^* = \frac{r_H \frac{c_H}{1-\eta} - 2r_H(1-r_2(w_2^*))p'd + (r_2(w_2^*) - r_H)p}{r_2(w_2^*)}$ (i.e., the highest price that allows manufacturer 2 to receive d from each buyer).

- Suppose $w_2^* \leq \bar{w}$ (which implies $r_2 = r_L$). Then, we have
$$w_2^* = \frac{r_H \frac{c_H}{1-\eta} - 2r_H(1-r_L)p'd + (r_L - r_H)p}{r_L} < \frac{c_L}{1-\eta},$$
where the last inequality follows from $c_H \leq \widehat{c}_H$, i.e., $\frac{r_H c_H - r_L c_L}{1-\eta} \leq -2(r_H + 2r_L - 3r_H r_L)p'd + (r_H - r_L)p < 2r_H(1-r_L)p'd + (r_H - r_L)p$. This contradicts with $w_2 \geq \frac{c_L}{1-\eta}$.

- Suppose $w_2^* > \bar{w}$ (which implies $r_2 = r_H$). Then, we have

$$w_2^* = \frac{r_H \frac{c_H}{1-\eta} - 2r_H(1-r_H)p'd}{r_H} < \frac{c_H}{1-\eta},$$

  which contradicts with $w_2^* > \bar{w}$.

Therefore, there must be no equilibrium prices that lead to $Q_{1j}^* = 0$ and $Q_{2j}^* = d$.

(ii) Suppose we have $Q_{1j}^* = d$ and $Q_{2j}^* = 0$ in equilibrium. Then, must have $w_2^* = \frac{c_L}{1-\eta} < \bar{w}$, which implies $r_2 = r_L$. Thus, the equilibrium prices must be given by $w_1 = \frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H}$, $w_2 = \frac{c_L}{1-\eta}$. We next prove that these are equilibrium prices.

First, we prove that fixing $w_1 = \frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H}$, manufacturer 2 will not deviate from $w_2 = \frac{c_L}{1-\eta}$. Clearly, deviating to a price such that $\frac{c_L}{1-\eta} < w_2 \leq \bar{w}$ leads to a zero profit due to a zero demand from buyers. Moreover, deviating to a price such that $w_2 > \bar{w}$ (which implies $r_2 = r_H$) also leads to a zero profit due to a zero demand from buyers, as it violates the second inequality in B.3. Under Assumption B.1, we have

$$\begin{aligned} w_2 > \bar{w} &\geq \frac{r_L \frac{c_L}{1-\eta} + 2(r_H + r_L)(1-r_L)p'd + (r_H - r_L)p}{r_H} \\ &> \frac{r_L \frac{c_L}{1-\eta} + 2(r_H - r_L)(1-r_H)p'd + (r_H - r_L)p}{r_H} \\ &= \frac{r_H \frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H} + 2r_H(1-r_H)p'd}{r_H}. \end{aligned}$$

Second, we prove fixing $w_2 = \frac{c_L}{1-\eta}$, manufacturer 1 will not have an incentive to deviate from $w_1 = \frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H}$. This price is the lower bound of w_1 in condition B.3. Thus, to prove that manufacturer 1 does not have an incentive to deviate, it is sufficient to prove that the price that satisfies the first-order condition (i.e., the price given in Equation B.5) is lower than $\frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H}$. This is true because

$$\begin{aligned} &\frac{r_L \frac{c_L}{1-\eta} + r_H \frac{c_H}{1-\eta} + 2r_H(1-r_L)p'd + (r_H - r_L)p}{2r_H} - \frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H} \\ =&\frac{r_H \frac{c_H}{1-\eta} - r_L \frac{c_L}{1-\eta} + 2(r_H + 2r_L - 3r_H r_L)p'd - (r_H - r_L)p}{2r_H} \\ \leq& 0, \end{aligned}$$

where the inequality follows from $c_H \leq \widehat{c}_H$, i.e., $\frac{r_H c_H - r_L c_L}{1-\eta} \leq -2(r_H + 2r_L - 3r_H r_L)p'd + (r_H - r_L)p$. Therefore, $w_1^* = \frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H}$, $w_2^* = \frac{c_L}{1-\eta}$ are equilibrium prices.

(iii) Suppose we have $Q_{1j}^* > 0$ and $Q_{2j}^* > 0$ in equilibrium. Then, the equilibrium prices w_1^*, w_2^* must satisfy B.3. Further, given $r_1 = r_H$, w_1^* must satisfy the first order condition in Equation B.5, i.e.,

$$w_1^* = \frac{r_2(w_2^*)w_2^* + r_H\left(\frac{c_H}{1-\eta} + 2(1 - r_2(w_2^*))p'd\right) + (r_H - r_2(w_2^*))\,p}{2r_H}.$$

- Suppose $w_2^* \le \bar{w}$ (which implies $r_2 = r_L$). Then, we have

$$w_1^* = \frac{r_L w_2^* + r_H\left(\frac{c_H}{1-\eta} + 2(1-r_L)p'd\right) + (r_H - r_L)\,p}{2r_H}.$$

  We next show that w_1^* violates the first inequality in B.3. Since $w_2^* \ge \frac{c_L}{1-\eta}$, we have

$$\begin{aligned}
&\frac{r_L w_2^* + r_H\left(\frac{c_H}{1-\eta} + 2(1-r_L)p'd\right) + (r_H - r_L)\,p}{2r_H} - \frac{r_L w_2^* - 2r_L(1-r_H)p'd + (r_H - r_L)\,p}{r_H}\\
=&\frac{-r_L w_2^* + r_H\frac{c_H}{1-\eta} + 2(r_H + 2r_L - 3r_H r_L)p'd - (r_H - r_L)\,p}{2r_H}\\
\le&\frac{r_H\frac{c_H}{1-\eta} - r_L\frac{c_L}{1-\eta} + 2(r_H + 2r_L - 3r_H r_L)p'd - (r_H - r_L)\,p}{2r_H}\\
\le&0.
\end{aligned}$$

  where the last inequality follows from $c_H \le \widehat{c}_H$, i.e., $\frac{r_H c_H - r_L c_L}{1-\eta} \le -2(r_H + 2r_L - 3r_H r_L)p'd + (r_H - r_L)p$.

- Suppose $w_2^* > \bar{w}$ (which implies $r_2 = r_H$). Then, by solving Equation B.5 for $i = 1, 2$, we have

$$w_1^* = w_2^* = \frac{c_H}{1-\eta} + 2(1-r_H)p'd < \bar{w},$$

  where the inequality follows from Assumption B.1. This contradicts with $w_2^* > \bar{w}$.

Therefore, there must be no equilibrium prices that lead to $Q_{1j}^* > 0$ and $Q_{2j}^* > 0$.

To summarize, given that one manufacturer chooses certified high reliability while the other manufacturer does not get certified, we must have that in equilibrium, the certified high-reliability manufacturer receives a demand d from each buyer at the price of $\frac{r_L\frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H}$, and the non-certified manufacturer chooses r_L, sets its price at $\frac{c_L}{1-\eta}$, and receives nothing from each buyer.

Third, we characterize manufacturers' certification and reliability decisions. From the above analysis, we can see that in equilibrium, a non-certified manufacturer always chooses r_L. That is, manufacturers never choose non-certified high reliability in equilibrium. Therefore, it is sufficient to analyze whether each manufacturer chooses certified high reliability or non-certified low reliability. Let Π_i^{AB} denote the expected profit of manufacturer i given the decision A of manufacturer 1 and B of manufacturer 2, where $A, B \in \{C \text{ (certified high reliability)}, N \text{ (not certified low reliability)}\}$. Due to symmetry, it is sufficient to characterize manufacturer 1's optimal decision given manufacturer 2's decision. Consider the following two cases:

(i) Given that manufacturer 2 chooses non-certified low reliability, we have the following:

  (a) If manufacturer 1 chooses non-certified low reliability, based on our analysis above, it will receive $\frac{1}{2}d$ from each buyer at the price of $\frac{c_L}{1-\eta} + 2(1-r_L)p'd$.

  (b) If manufacturer 1 chooses certified high reliability, based on our analysis above, it will receive d from each buyer at the price of $\frac{r_L\frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H}$.

Therefore, we have

$$\begin{aligned}\Pi_1^{NN} - \Pi_1^{CN} =& (1-\eta)r_L\left(\frac{c_L}{1-\eta} + 2(1-r_L)p'd - \frac{c_L}{1-\eta}\right)\frac{D}{2} \\ & - (1-\eta)r_H\left(\frac{r_L\frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H} - \frac{c_H}{1-\eta}\right)D + f \\ =& (1-\eta)\left(\frac{r_Hc_H - r_Lc_L}{1-\eta} + \left(3r_L - r_L^2 - 2r_Hr_L\right)p'd - (r_H - r_L)p + \frac{f}{(1-\eta)D}\right)D.\end{aligned}$$

Thus, manufacturer 1 will choose certified high reliability given

$$\begin{aligned} & c_H \le \widehat{c}_H \le \frac{r_Lc_L - (1-\eta)\left(2r_L(2-r_H-r_L)p'd - (r_H-r_L)p\right) - \frac{f}{D}}{r_H} \\ \Longrightarrow & \frac{r_Hc_H - r_Lc_L}{1-\eta} \le -\left(3r_L - r_L^2 - 2r_Hr_L\right)p'd - r_L(1-r_L)p'd + (r_H - r_L)p - \frac{f}{(1-\eta)D} \\ \Longrightarrow & \frac{r_Hc_H - r_Lc_L}{1-\eta} < -\left(3r_L - r_L^2 - 2r_Hr_L\right)p'd + (r_H - r_L)p - \frac{f}{(1-\eta)D}.\end{aligned}$$

(ii) Given that manufacturer 2 chooses certified high reliability, we have the following:

(a) If manufacturer 1 chooses non-certified low reliability, based on our analysis above, it will receive nothing from each buyer.

(b) If manufacturer 1 chooses certified high reliability, based on our analysis above, it will receive $\frac{d}{2}$ from each buyer at the price of $\frac{c_H}{1-\eta} + 2(1-r_H)p'd$.

Therefore, we have

$$\begin{aligned}\Pi_1^{NC} - \Pi_1^{CC} =& 0 - (1-\eta)r_H\left(\frac{c_H}{1-\eta} + 2(1-r_H)p'd - \frac{c_H}{1-\eta}\right)\frac{D}{2} + f \\ =& -(1-\eta)r_H(1-r_H)p'dD + f \\ \ge& 0,\end{aligned}$$

where the last inequality follows from $r_H \ge \widehat{r}_H \ge \frac{1+\sqrt{1-\frac{4f}{(1-\eta)p'dD}}}{2}\mathbf{1}[1 > \frac{4f}{(1-\eta)p'dD}]$, which implies $r_H(1-r_H) \le \frac{f}{(1-\eta)p'dD}$. Thus, manufacturer 1 will choose non-certified low reliability.

Step 3: Finally, we compare the performance metrics with and without certification.

(i) Patients' shortage cost: With certification, based on our analysis in Step 2, we know that one manufacturer chooses r_H while the other manufacturer chooses r_L; buyers source a total of D from the high-reliability manufacturer and nothing from the low-reliability manufacturer. Then, there is a probability $1-r_H$ of experiencing a shortage quantity of D, leading to an expected shortage cost of $\gamma(1-r_H)D^2$. Without certification, based on our analysis in Step 1, both manufacturers choose r_L, and buyers source $\frac{D}{2}$ from each manufacturer. Then, there is a probability $2r_L(1-r_L)$ of experiencing a shortage quantity of $\frac{D}{2}$ (i.e., when exactly one manufacturers is under disruption) and a probability $(1-r_L)^2$ of experiencing a shortage quantity of D (i.e., when both manufacturers are under disruption). Therefore, in this case, the expected shortage cost is given by $2\gamma r_L(1-r_L)(\frac{D}{2})^2 + \gamma(1-r_L)^2D^2 = \gamma\frac{r_L^2-3r_L+2}{2}D^2$.

To conclude, certification leads to a lower expected shortage cost compared to no certification given $r_H > \widehat{r}_H \geq \frac{3}{2}r_L - \frac{1}{2}r_L^2 \implies 1 - r_H < \frac{r_L^2 - 3r_L + 2}{2}$.

(ii) Expected manufacturer's profit: With certification, based on our analysis in Step 2, one manufacturer chooses r_H, and the high-reliability manufacturer receives a demand D from the buyers at the price of $\frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H}$, while the low-reliability manufacturer receives nothing. Then, the expected average manufacturer profit is

$$\frac{1}{2}\left(r_H\left((1-\eta)\frac{r_L\frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H} - c_H\right)D - f\right)$$
$$= (r_L c_L - r_H c_H - 2(1-\eta)r_L(1-r_H)p'd + (1-\eta)(r_H - r_L)p)\frac{D}{2} - \frac{f}{2}.$$

Without certification, based on our analysis in Step 1, each manufacturer chooses r_L and receives $\frac{D}{2}$ at the price of $\frac{c_L}{1-\eta} + 2(1-r_L)p'd$. Then, the expected average manufacturer profit is $(1-\eta)r_L(1-r_L)p'dD$.

To conclude, certification leads to a higher expected manufacturer's profit given

$$c_H \leq \widehat{c}_H \leq \frac{r_L c_L - (1-\eta)\left(2r_L(2 - r_H - r_L)p'd - (r_H - r_L)p\right) - \frac{f}{D}}{r_H}$$
$$\implies \left(r_L c_L - r_H c_H - 2(1-\eta)r_L(1-r_H)p'd + (1-\eta)(r_H - r_L)p\right)\frac{D}{2} - \frac{f}{2} \geq (1-\eta)r_L(1-r_L)p'dD.$$

(iii) Expected buyer cost: With certification, based on our analysis in Step 2, one manufacturer chooses r_H and one manufacturer chooses r_L. Each buyer source d from the high-reliability manufacturer at the price of $\frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H}$ and nothing from the low-reliability manufacturer. There is a probability r_H that each buyer pays the high-reliability manufacturer for the delivered quantity d, and a probability $1 - r_H$ of paying a shortage penalty $pd + p'd^2$. This leads to an expected buyer cost of $r_H \frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H} d + (1-r_H)(pd + p'd^2) = -(r_H + 2r_L - 2r_H r_L - 1)p'd^2 + (1 - r_L)pd + r_L\frac{c_L}{1-\eta}d$. Without certification, based on our analysis in Step 1, both manufacturer chooses r_L, and each buyer source $\frac{d}{2}$ from each manufacturer at the price of $\frac{c_L}{1-\eta} + 2(1-r_L)p'd$. This leads to an expected buyer cost of

$$r_L^2\left(\frac{c_L}{1-\eta} + 2(1-r_L)p'd\right)d + 2r_L(1-r_L)\left(\frac{c_L}{1-\eta} + 2(1-r_L)p'd + p + \frac{p'd}{2}\right)\frac{d}{2} + (1-r_L)^2(p + p'd)d$$
$$= (1-r_L)\left(1 + \frac{3}{2}r_L\right)p'd^2 + (1-r_L)pd + r_L\frac{c_L}{1-\eta}d.$$

Therefore, certification leads to a lower expected buyer cost because

$$-(r_H + 2r_L - 2r_H r_L - 1)p'd^2 - (1-r_L)\left(1 + \frac{3}{2}r_L\right)p'd^2 = \left(-r_H - \frac{5}{2}r_L + 2r_H r_L + \frac{3}{2}r_L^2\right)p'd^2 < 0.$$

(iv) Expected GPO profit: With certification, based on our analysis in Step 2, one manufacturer chooses r_H, and the high-reliability manufacturer has a probability of r_H delivering D demands to the buyers at the price of $\frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H}$, leading to an expected revenue of $r_H \frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H} D$ for the manufacturer. On the other hand, the low-reliability manufacturer receives nothing from the buyers. Therefore, the expected GPO profit is given by

$$\eta r_H \frac{r_L \frac{c_L}{1-\eta} - 2r_L(1-r_H)p'd + (r_H - r_L)p}{r_H} D = -2\eta r_L(1-r_H)p'dD + \eta(r_H - r_L)pD + \eta r_L \frac{c_L}{1-\eta} D.$$

Without certification, based on our analysis in Step 1, we have that each manufacturer chooses r_L, and with probability r_L, delivers $\frac{D}{2}$ to the buyers at the price $\frac{c_L}{1-\eta} + 2(1-r_L)p'd$, leading to an expected GPO profit of $2\eta r_L(\frac{c_L}{1-\eta} + 2(1-r_L)p'd)\frac{D}{2} = 2\eta r_L(1-r_L)p'dD + \eta r_L \frac{c_L}{1-\eta} D$.

Therefore, certification leads to a higher expected GPO profit given

$$r_H \geq \widehat{r}_H \geq \frac{p + 2(2-r_L)p'd}{p + 2r_L p'd} r_L$$

$$\Longrightarrow r_H(p + 2r_L p'd) \geq r_L p + 2r_L(2-r_L)p'd$$

$$\Longleftrightarrow -2\eta r_L(1-r_H)p'dD + \eta(r_H - r_L)pD + \eta r_L \frac{c_L}{1-\eta} D \geq 2\eta r_L(1-r_L)p'dD + \eta r_L \frac{c_L}{1-\eta} D.$$

Therefore, the conclusion of the proposition holds. □

C Convex Production Cost for Manufacturers

In our main model, for ease of exposition, we assume that each manufacturer's cost is linear in its reliability level. To demonstrate the robustness of our insights, we now extend our model to incorporate convex production costs to capture the potentially increasing marginal cost of improving reliability. Specifically, for a manufacturer with reliability level r_i, we now consider a unit production cost $a(r_i - r_L) + b(r_i - r_L)^2 + c_L$, where $b \geq 0$.

Since we focus on a practical setting where the reliability improvement cost is relatively high (i.e., Assumption 1), we have shown under our main model that, without certification, manufacturers retain the baseline reliability level r_L. As each manufacturer's production cost now includes an additional positive term, it is straightforward to verify that this conclusion continues to hold. The next lemma characterizes the equilibrium decisions of all players under certification. For notational convenience, we define $a' = \bar{\bar{a}} - b(r_H - r_L)$ in this section.

Lemma C.1. *With certification, the equilibrium decisions of all players are characterized as follows:*
(i) If $a \geq a'$, neither manufacturer improves reliability. Further, if $c_L \leq \bar{\bar{c}}_L(r_L)$, then both manufacturers set a wholesale price of $(1-r_L)p + r_L s$, and each buyer sources d from each manufacturer; otherwise (i.e., $c_L > \bar{\bar{c}}_L(r_L)$), both manufacturers set a wholesale price of $\frac{c_L}{1-\eta}$, and each buyer sources $\frac{1}{2}d$ from each manufacturer.
(ii) If $a < a'$, one manufacturer improves its reliability to r_H and sets a wholesale price of $\frac{r_L \frac{c_L}{1-\eta} + (r_H - r_L)p}{r_H} - \epsilon$, while the other manufacturer retails at low reliability r_L and sets a wholesale price of $\frac{c_L}{1-\eta}$. Each buyer single sources d from the high reliability manufacturer.

Lemma C.1 shows that with certification, the equilibrium outcomes remain structurally the same as in our main model, except that the threshold on a above which neither manufacturer improves reliability is lower due to the additional positive quadratic term in each manufacturer's production cost. The next proposition further shows that certification leads to a Pareto improvement under similar conditions as in our main model.

Proposition C.1. *When each manufacturer's production cost is convex to its reliability level, certification leads to a Pareto improvement for patients, manufacturers, buyers, and the GPO if* $r_H \geq \bar{r}_H$, $a \leq \bar{a} - b(r_H - r_L)$ *and* $c_L \geq \bar{c}_L$.

Consistent with Corollary 1, Proposition A.2 shows that certification benefits all stakeholders when (1) the certification threshold for high reliability (i.e., r_H) is sufficiently high; (2) reliability is not too costly (i.e., a is not too high); and (3) the production cost before reliability improvement (i.e., c_L) is relatively high such that manufacturer profit margins are low. This result demonstrates that our key insights derived from the main model continue to hold when each manufacturer's production cost increases convexly with its reliability level.

C.1 Proofs

Proof of Lemma C.1. The proof of this lemma is similar to that of Lemma 3. First, Step 1 of the proof remains valid here because each buyer's decisions and cost function are the same as before. Second, in Step 2 of the proof, for each certification status profile (δ_1, δ_2) that may arise in equilibrium, we have derived necessary conditions that the corresponding equilibrium reliability levels and prices must satisfy. Specifically, we show that for $i \in \{1, 2\}$, in equilibrium, if $\delta_i = 0$, we must have $r_i = r_L$; if $\delta_i = 1$, we must have $r_i = r_H$. Since each manufacturer's product cost now includes an additional positive term, manufacturers will not have the incentive to choose higher reliability levels, and thus our conclusion in Step 2 of the proof of Lemma 3 also continues to hold.

Therefore, to characterize the equilibrium, it remains to check how Step 3 of the proof of Lemma 3 needs to be revised. As in the proof of Lemma 3, let $\Pi_i(\delta_1, \delta_2)$ denote the manufacturer i's expected profit given the certification status profile (δ_1, δ_2), where the corresponding reliability and pricing choices satisfy the equilibrium conditions characterized in Step 2 of the proof of Lemma 3.

Recall that we have defined $c_H = c_L + a(r_H - r_L)$ in the proof of Lemma 3 as the production cost when a manufacturer chooses a reliability level of r_H. Let $c'_H = c_L + a(r_H - r_L) + b(r_H - r_L)^2$. Then, Step 3 of the proof of Lemma 3 continues to hold by replacing the production cost c_H with c'_H. Since

$$c'_H < \bar{\bar{c}}_H = \min\{\frac{2r_L c_L + (1-\eta)\left(\left(r_L^2 - 2r_L + r_H\right)p - r_L^2 s\right)}{r_H}, \frac{r_L c_L + (1-\eta)(r_H - r_L)p}{r_H}\}$$

$$\iff c_L + a(r_H - r_L) + b(r_H - r_L)^2 < c_L + \bar{\bar{a}}(r_H - r_L)$$

$$\iff a < \bar{\bar{a}} - b(r_H - r_L),$$

we conclude that when $a < \bar{\bar{a}} - b(r_H - r_L)$, one manufacturer chooses a reliability level of r_H while the other chooses a reliability level of r_L in equilibrium. Otherwise, neither manufacturer improves reliability. □

Proof of Proposition C.1. As discussed at the beginning of Supplemental Appendix C, the equilibrium outcomes without certification remain unchanged from those under our main model (see Lemma 2). Then, we can prove this proposition by following similar steps to the proof of Corollary 1, using the equilibrium outcomes without certification from Lemma 2 and those with certification from Lemma C.1. Specifically, from the analysis in the proof of Lemma C.1, we conclude that the proof of Corollary 1 continues to hold by replacing the production cost $c_H = c_L + a(r_H - r_L)$ with $c'_H = c_L + a(r_H - r_L) + b(r_H - r_L)^2$.

Recall from our proof of Proposition 2 that when one manufacturer chooses r_H while the other chooses r_L under certification, certification leads to a lower expected manufacturer profit if and only if

$$c_H \geq \frac{r_L c_L + (1-\eta)(r_H - r_L)p - \frac{2f}{D} - 2r_L(\bar{\bar{c}}_L(r_L) - c_L)^+}{r_H} \iff a \geq \bar{a}.$$

With a convex production cost, the above condition needs to be revised to

$$c'_H \geq \frac{r_L c_L + (1-\eta)(r_H - r_L)p - \frac{2f}{D} - 2r_L(\bar{\bar{c}}_L(r_L) - c_L)^+}{r_H} \iff a \geq \bar{a} - b(r_H - r_L).$$

Then, following the analysis in the proof of Corollary 1, we conclude that certification leads to a Pareto improvement for all stakeholders if $r_H \geq \bar{r}_H$, $a \leq \bar{a} - b(r_H - r_L)$ and $c_L \geq \bar{c}_L$. □